

Master's Thesis

Regulatory Communication, Investor Networks & Bank Runs

Aditya Srivastava

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Supervisors

Dr. Kartik Anand
Deutsche Bundesbank

Prof. Jean-Edouard Colliard
HEC Paris

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Abstract

This thesis studies how the structure and communication of information shape financial coordination when investors make decisions under uncertainty and their actions depend on expectations about the behavior of others. Building on the global-games framework, I develop two models of information and coordination in which investors form beliefs about an institution's underlying financial condition using both private and publicly available information.

The first model studies coarse regulatory communication. A regulator observes a noisy signal of the institution's financial condition but communicates only a binary assessment rather than revealing its information in full. This captures settings in which complex regulatory assessments, such as stress tests or communication during periods of financial stress, are condensed into simple public messages. I characterize the resulting threshold equilibrium and show that coarse communication can preserve equilibrium uniqueness even when the regulator's underlying information becomes arbitrarily precise. The result highlights a distinction between the precision of the regulator's information and the information conveyed through its communication: a perfectly informed regulator need not eliminate strategic uncertainty when its communication remains sufficiently coarse.

The second model studies coordination when information is also transmitted through a network. Investors observe signals from their neighbours alongside an exogenous public signal from the regulator, generating heterogeneous information sets and uncertainty about what others know. I show that more precise public information attenuates the influence of decentralized information and reduces higher-order uncertainty, thereby changing the informational role of the network. This mechanism is particularly relevant to episodes of financial stress, such as the Silicon Valley Bank crisis, in which information and beliefs spread rapidly through social and professional networks alongside publicly available signals.

Together, the two models show that the consequences of information for coordination depend not only on how precise that information is, but also on how it is communicated and how it is distributed across investors.

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1 Introduction

Financial crises are often driven not only by the underlying condition of financial institutions, but also by expectations about how others will respond to that condition. A bank may experience rising losses, deteriorating asset quality, or increasing liquidity pressure without immediately becoming insolvent. Yet concerns about its financial weakness can become self-reinforcing when depositors expect other depositors to withdraw. The decision to keep funds in the bank is therefore both a judgment about the institution's underlying condition and a coordination problem: even an institution that remains viable under normal circumstances may become vulnerable when sufficiently many depositors expect others to run.

This interaction between fundamentals and expectations makes information and communication central to financial stability. Depositors, investors, and other market participants rarely observe a bank's underlying condition perfectly. Instead, they form views from dispersed and noisy sources of information while also trying to infer what others know and how they are likely to react. Public communication by regulators and central banks enters this environment in a particularly important way. Supervisory authorities may possess information about an institution's financial condition that is not directly available to market participants and may communicate their assessments through stress tests, supervisory statements, or other public announcements. Such communication can therefore affect coordination not only by changing beliefs about the institution's underlying condition, but also by changing beliefs about the information and expected actions of others.

This thesis studies these questions within a global-games framework. The common fundamental is interpreted as the underlying financial weakness of an institution: higher values correspond to a more vulnerable institution and make keeping funds in the institution less attractive. Investors receive noisy information about this condition and choose between maintaining their position and withdrawing to a safe alternative. Their decisions are strategically linked because the payoff from maintaining their position also depends on the behaviour of other investors. The central question is how regulatory communication shapes this coordination problem when information is dispersed and investors are uncertain not only about the fundamental, but also about what others know.

The thesis develops two models that study different, but closely related, dimensions of this financial coordination problem. The first asks what happens when a regulator possesses detailed information about an institution's financial condition but communicates only a coarse assessment. Rather than observing the regulator's signal directly, investors learn only whether the regulatory assessment lies on one side or the other of a cutoff. The cutoff can be interpreted as a supervisory benchmark separating relatively strong from relatively weak assessments of the institution's condition. This captures an important feature of regulatory communication: a stress test or supervisory assessment may be based

on a rich set of information, while its public communication compresses that information into a relatively simple outcome, classification, or summary.

The distinction between the precision of the regulator's information and the precision of its communication is central to the first model. A regulator may observe an institution's financial weakness with very high precision while investors receive only a binary assessment. Coarse communication can therefore preserve uncertainty about the exact state of the institution and, more importantly, about the beliefs and actions of other investors. I characterize the resulting threshold equilibrium and establish conditions for existence and uniqueness. A central result is that equilibrium uniqueness can survive even as the regulator's underlying information becomes arbitrarily precise. Thus, the implications of public information depend not only on how accurately the regulator observes the fundamental, but also on how that information is aggregated and communicated to investors.

The second model considers the same coordination problem in an environment in which information also spreads through a network. Investors receive private information about the institution's financial condition and observe the signals available to their neighbours, alongside a public signal from the regulator. The resulting information sets are heterogeneous and overlapping: an investor may know something about the institution that another investor does not, while also being uncertain about the information on which that investor is basing her decision. This generates higher-order uncertainty that depends on the architecture of information transmission. The setting is particularly relevant to modern bank runs, such as the failure of Silicon Valley Bank, where information and concerns about financial conditions spread rapidly through concentrated social and professional networks alongside publicly available information and official communication.

The network model therefore examines how regulatory communication interacts with the decentralized channels through which investors learn from one another. I show that more precise public information attenuates the influence of decentralized private information and reduces uncertainty about what other investors know. Regulatory communication consequently affects coordination not only by improving investors' estimates of the underlying financial condition, but also by reshaping the higher-order beliefs through which expectations are formed. The analysis further shows that greater network connectivity need not have a mechanical effect: additional connections may reduce higher-order uncertainty when they reinforce information that is already shared, but may have different effects when they introduce new information that remains locally unobserved.

Taken together, the two models study a common question from two complementary perspectives. The first concerns *what* a regulator communicates: a supervisory authority may possess precise information about financial weakness while disclosing only a coarse assessment. The second concerns the informational environment into which that communication is released: even when regulatory information is public, investors may continue to learn through private social, professional, or financial networks. The broader implication

is that regulatory communication should not be understood simply as the transmission of information about fundamentals. By determining what investors learn, what remains decentralized, and what they can infer about the information of others, communication can shape the expectations and coordination dynamics through which financial fragility emerges.

1.1 Related Literature

This thesis builds on the literature on global games, public information and regulatory disclosure, and information aggregation through networks. Global games, originating with Carlsson and van Damme (1993) and developed by Morris and Shin (1998, 2000, 2001), provide a tractable framework for studying coordination problems in which agents face both uncertainty about an underlying fundamental and strategic uncertainty about the actions of others. A central insight is that sufficiently small amounts of private noise can eliminate the multiplicity associated with the corresponding complete-information coordination game and select a unique equilibrium. The framework has subsequently been applied to a range of financial coordination problems, including currency attacks (Morris and Shin, 1998), and bank runs (Goldstein and Pauzner, 2005). A particularly important extension introduces public information alongside dispersed private signals. Morris and Shin (2002) show that public information affects coordination not only by improving beliefs about the fundamental, but also by shaping higher-order beliefs because it is commonly observed. This insight has motivated a broader literature on the consequences of transparency and public disclosure, including Angeletos and Pavan (2007); Svensson (2006); Hellwig (2005); Morris et al. (2006). The present thesis builds on this perspective but focuses more specifically on how the form and precision of regulatory communication affect equilibrium uniqueness and coordination.

The first part of the thesis is closely related to the literature on regulatory disclosure, stress testing, and information design. Stress tests provide a natural setting in which a regulator possesses information that is not directly available to market participants and must decide how that information is communicated publicly. Goldstein and Sapra (2014) emphasize the tension between the informational benefits of disclosure and the possibility that public communication may itself affect coordination. Related work studies the design and consequences of stress-test disclosure and regulatory information in financial systems (Bouvard et al., 2015; Goldstein and Leitner, 2018; Corona et al., 2019; Pereira, 2021; Dai et al., 2024). These contributions highlight that regulatory disclosure can affect financial stability not simply by reducing uncertainty about fundamentals, but also by shaping expectations about the beliefs and actions of other market participants. This mechanism is particularly close to the first model in this thesis, where the regulator's message changes the informational environment in which agents coordinate. Related work on information design in global games with applications to stress testing is provided by

Inostroza and Pavan (2025), while the broader literature on Bayesian persuasion studies how an informed sender can design information to influence the behavior of receivers (Kamenica and Gentzkow, 2011).

In contrast to this literature, the communication technology in this thesis is taken as exogenous. The first model considers a regulator who observes a continuous noisy signal about the fundamental but communicates only a binary message. This distinction is consequential: the message reveals only the region in which the regulator's signal lies, generating non-Gaussian posterior beliefs even when the underlying information structure is Gaussian. The analysis draws on the skew-normal family of distributions, whose origins trace back to O'Hagan and Leonard (1976) and whose formal treatment was developed by Azzalini (2013). More closely related to the present setting, Avoyan (2024) studies coarse information in a two-player global game. The present model extends this approach to a coordination problem with n investors and uses the resulting posterior structure to characterize equilibrium thresholds and derive conditions for equilibrium uniqueness. The analysis therefore emphasizes not only how coarse regulatory communication generates non-Gaussian beliefs, but also how the precision of the regulator's underlying assessment can differ from the informational precision of the communication received by investors.

The second part of the thesis connects the global-games framework to the literature on networked information. In these models, agents do not acquire information independently but may observe or exchange signals through social, professional, or financial networks. Dahleh et al. (2012) and Mahdavifar et al. (2017) study global games with explicit information exchange and show how network structure and noisy communication affect threshold behavior and equilibrium uniqueness. de Martí and Milán (2019) consider information networks in which agents truthfully share private signals with their neighbours, emphasizing that network communication both aggregates information and creates correlations in posterior beliefs. Siemaszko (2024) further studies global games on social networks, focusing on the role of network topology and information-sharing incentives in equilibrium uniqueness and comparative statics. More broadly, recent empirical work by Cookson et al. (2026) highlights the importance of information transmission through decentralized social and financial networks for collective economic outcomes.

These contributions are particularly relevant because network structure affects coordination through beliefs and higher-order beliefs rather than through direct payoff externalities. The network model in this thesis focuses more specifically on informational overlap between agents. An agent's uncertainty about another's posterior depends not only on the size of the network, but on which signals are shared between their respective informational neighbourhoods. This makes it possible to distinguish network links that reinforce information already indirectly observed by other agents from links that introduce genuinely new and locally unshared information.

The network literature and the disclosure literature have, however, largely developed

separately. Existing networked global games study decentralized information exchange, while models of regulatory disclosure typically consider a regulator communicating directly with agents. The present thesis combines these two information structures. Agents may learn from their neighbours while simultaneously receiving a common signal from a regulator, allowing the analysis to examine how centralized communication interacts with decentralized information transmission. The two parts of the thesis therefore study complementary dimensions of the informational architecture of coordination: first, what the regulator reveals, and second, how public information interacts with the networks through which agents learn about one another.

2 Benchmark Global Game with Exogenous Public Signal

We consider a benchmark model of financial coordination. Consider a financial institution whose underlying condition is summarized by a common fundamental θ. Throughout the thesis, a higher value of θ corresponds to greater financial weakness. This may be interpreted broadly as reflecting, for example, deterioration in asset quality, expected credit losses, or other sources of balance-sheet fragility.

A population of depositors or short-term investors must decide whether to withdraw their funds or keep them invested in the institution. This decision depends both on the institution's underlying condition and on expectations about the decisions of other investors. Widespread withdrawals can amplify financial distress, whereas continued funding by other investors makes it more attractive to remain invested. The resulting strategic complementarity gives rise to a coordination problem.

The benchmark global game assumes that agents receive noisy private information about the institution's underlying condition and additionally observe an exogenous public signal from a regulator. The regulator is modeled solely as a source of public information; we therefore do not specify a regulatory objective or an optimal communication problem.

2.1 Information Structure and Complete-Information Benchmark

Consider n agents, indexed by $i = 1, \ldots, n$. The fundamental θ is normally distributed according to

$$\theta \sim N\left(\bar{\theta}, \frac{1}{\alpha}\right), \tag{1}$$

where α denotes the precision of the prior. Each investor chooses $a_i \in \{0, 1\}$. Choosing $a_i = 0$ corresponds to withdrawing funds from the institution and obtaining a safe payoff normalized to zero. Choosing $a_i = 1$ corresponds to keeping funds invested and remaining

exposed to the institution's underlying condition. Let

$$k = \sum_{j=1}^{n} a_j$$

denote the number of investors who keep their funds invested. Investor i's payoff is

$$u_i(a_i, k, \theta) = a_i \left(\frac{k}{n} - \theta \right). \tag{2}$$

The payoff is normalized so that withdrawing yields zero. Conditional on remaining invested, the payoff decreases with the institution's financial weakness and increases with the fraction of investors who also keep their funds in the institution. The term k/n should therefore be interpreted as a reduced-form benefit of continued collective funding and confidence, while θ captures the underlying weakness of the institution.

It is useful to first consider the complete-information version of the game, in which the financial weakness θ is common knowledge.

Suppose first that all other $n-1$ agents choose the safe action, $a_j = 0$, corresponding to withdrawing their funds. If agent i deviates and instead keeps their funds invested, their payoff is

$$\frac{1}{n} - \theta. \tag{3}$$

Thus, keeping funds invested is optimal only if

$$\frac{1}{n} - \theta \geq 0, \tag{4}$$

or equivalently if $\theta \leq 1/n$. Hence, when $\theta > 1/n$, no agent wishes to deviate from the profile in which everyone withdraws, so the all-withdrawal profile is an equilibrium.

Conversely, suppose that all other $n-1$ agents keep their funds invested, so that $a_j = 1$. Agent i then obtains

$$1 - \theta \tag{5}$$

from also keeping its funds invested. This is optimal whenever

$$1 - \theta \geq 0, \tag{6}$$

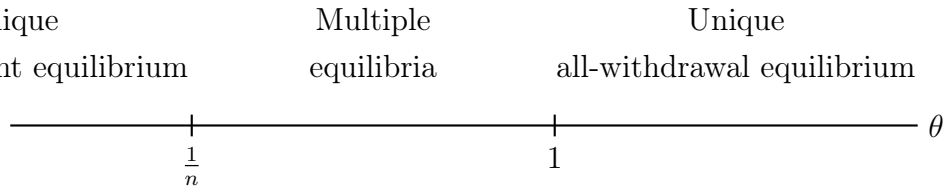
or equivalently whenever $\theta \leq 1$. Hence, for $\theta < 1$, the all-investment profile is an equilibrium.

The complete-information game therefore has the following structure: for sufficiently low levels of financial weakness, $\theta < 1/n$, keeping funds invested is optimal even if all other investors withdraw, yielding a unique all-investment equilibrium. For sufficiently high levels of financial weakness, $\theta > 1$, withdrawing is optimal even if all other investors

remain invested, yielding a unique all-withdrawal equilibrium. For intermediate values,

$$\frac{1}{n} < \theta < 1, \tag{7}$$

both action profiles can be sustained as equilibria. The source of this multiplicity is strategic complementarity: whether it is optimal to keep funds invested depends on whether other investors do so as well.



We now introduce dispersed information. Each agent observes a noisy private signal about the institution's underlying financial weakness,

$$x_i = \theta + \varepsilon_i, \qquad \varepsilon_i \sim N\left(0, \frac{1}{\beta}\right), \tag{8}$$

where the signal noises are independent across investors and independent of θ. We assume that all investors' private signals have the same precision β. Thus,

$$\sigma_\theta^2 = \frac{1}{\alpha}, \qquad \sigma_x^2 = \frac{1}{\beta}.$$

In addition, all agents observe a common public signal generated by the regulator,

$$y = \theta + \varepsilon_R, \qquad \varepsilon_R \sim N\left(0, \frac{1}{\gamma}\right), \tag{9}$$

The signal may be interpreted as a public regulatory assessment of the institution's financial condition. In the benchmark, this assessment is disclosed directly and in full, so that all investors observe the same realization y.

2.2 Threshold Equilibrium

We restrict attention to symmetric monotone threshold equilibria. In such an equilibrium, for each realization of the public regulatory signal y, there exists a threshold $t(y)$ such that

$$a_i = 1 \quad \Longleftrightarrow \quad x_i \leq t(y). \tag{10}$$

An agent remains invested when its private signal indicates sufficiently low financial weakness. A higher private signal raises the agent's posterior belief that the institution is weak and therefore makes remaining invested less attractive.

To characterize the threshold, consider an agent i who observes (x_i, y). The posterior distribution of the fundamental is

$$\theta \mid x_i, y \sim N\left(\mu_i^P, \frac{1}{\alpha+\beta+\gamma}\right), \tag{11}$$

where

$$\mu_i^P = \frac{\alpha\bar{\theta} + \beta x_i + \gamma y}{\alpha+\beta+\gamma}. \tag{12}$$

Let $k_{-i} = \sum_{j\neq i} a_j$ denote the number of other agents who keep their funds invested. Given (x_i, y), the expected payoff from remaining invested is

$$E\left[\frac{1+k_{-i}}{n} - \theta \,\middle|\, x_i, y\right]. \tag{13}$$

Hence, agent i remains invested whenever

$$\frac{1}{n} + \frac{1}{n}E[k_{-i} \mid x_i, y] - \mu_i^P \geq 0. \tag{14}$$

Under the threshold strategy in (10), another agent j chooses to keep their funds invested if and only if $x_j \leq t(y)$. To evaluate agent i's expectation of a_j, we therefore require the conditional distribution of x_j given (x_i, y). Since

$$x_j = \theta + \varepsilon_j,$$

and ε_j is independent of (x_i, y) conditional on θ, we obtain

$$x_j \mid x_i, y \sim N\left(\mu_i^P, \frac{1}{\beta} + \frac{1}{\alpha+\beta+\gamma}\right). \tag{15}$$

Consequently,

$$\Pr(a_j = 1 \mid x_i, y) = \Phi\left(\frac{t(y) - \mu_i^P}{\sqrt{\frac{1}{\beta} + \frac{1}{\alpha+\beta+\gamma}}}\right). \tag{16}$$

By symmetry,

$$E[k_{-i} \mid x_i, y] = (n-1)\Phi\left(\frac{t(y) - \mu_i^P}{\sqrt{\frac{1}{\beta} + \frac{1}{\alpha+\beta+\gamma}}}\right). \tag{17}$$

Substituting (17) into the best-response condition gives

$$\frac{1}{n} + \frac{n-1}{n}\Phi\left(\frac{t(y) - \mu_i^P}{\sqrt{\frac{1}{\beta} + \frac{1}{\alpha+\beta+\gamma}}}\right) - \mu_i^P \geq 0. \tag{18}$$

The threshold $t(y)$ is the private signal at which agent i is indifferent between the two

actions. Define the associated threshold for the posterior mean of the fundamental by

$$t^*(y) \equiv \mu_i^P(t(y), y). \tag{19}$$

Using (12),

$$t^*(y) = \frac{\alpha\bar{\theta} + \beta t(y) + \gamma y}{\alpha + \beta + \gamma}, \tag{20}$$

and hence

$$t(y) = \frac{(\alpha + \beta + \gamma)t^*(y) - \alpha\bar{\theta} - \gamma y}{\beta}. \tag{21}$$

At the threshold, (18) holds with equality. Substituting (21) and using $\mu_i^P = t^*(y)$ yields the equilibrium threshold equation

$$t^*(y) = \frac{1}{n} + \frac{n-1}{n}\Phi\left(\frac{(\alpha+\gamma)t^*(y) - \alpha\bar{\theta} - \gamma y}{\beta\sqrt{\frac{1}{\beta} + \frac{1}{\alpha+\beta+\gamma}}}\right). \tag{22}$$

The existence and uniqueness properties of this threshold equilibrium are established in Proposition 1; the proof is provided in Appendix A.

2.3 Existence and Uniqueness

The threshold equation admits a solution for every realization of the public signal. Moreover, the following proposition characterizes when this solution is unique.

Proposition 1 (Existence and uniqueness)**.** *For every realization of the public signal y, there exists a monotone threshold equilibrium. This equilibrium is unique for every y if and only if*

$$\frac{(\alpha+\gamma)\sqrt{\alpha+\beta+\gamma}}{\sqrt{\beta}\sqrt{\alpha+2\beta+\gamma}} \leq \frac{n}{n-1}\sqrt{2\pi}. \tag{23}$$

The proof is provided in Appendix A. The condition captures the standard global-game tension associated with public information. A more precise regulatory assessment improves agents' information about the institution's underlying condition, but because the assessment is observed by everyone, it also makes beliefs and expectations more closely aligned. When regulatory information becomes sufficiently precise, this coordinating force can dominate the dispersion generated by private information, and equilibrium uniqueness may break down.

In particular, we have

$$\sigma_R \to 0 \quad \Longleftrightarrow \quad \gamma \to \infty,$$

and the left-hand side of (23) diverges. Thus, uniqueness eventually fails as the public signal becomes arbitrarily precise.

Finally, as private information becomes arbitrarily precise, $\sigma_x \to 0$ (equivalently, $\beta \to \infty$), the selected threshold converges to

$$t^*(y) \longrightarrow \frac{n+1}{2n}. \tag{24}$$

For a large population, this converges to 1/2. The benchmark therefore provides the baseline information structure for the rest of the thesis: agents combine dispersed private information about the institution's financial condition with a common regulatory assessment that is observed publicly and in full. The precision of this assessment affects both beliefs about financial weakness and higher-order beliefs about what other agents know.

3 Coarse Regulatory Communication

We now relax the assumption that regulatory information is communicated directly and in full. This distinction is important because the information available to a regulator need not coincide with the information ultimately available to investors. A supervisory authority may form a detailed assessment of an institution's financial condition while communicating only a simple classification or verdict. In such a setting, making the regulator better informed does not necessarily make investors equally well informed: the additional information may remain hidden behind the communication rule.

This raises a different question from the benchmark. How does coordination change when the regulator observes the fundamental with increasing precision, but investors receive only a coarse summary of that assessment? The key issue is therefore not only how much the regulator knows, but how its information is transmitted to investors. The next section studies this case and examines its implications for equilibrium existence, uniqueness, and coordination.

3.1 Regulator's Signal and Binary Message

The underlying environment remains the same as in the benchmark: the fundamental is uncertain, investors receive noisy private signals, and their decisions are strategically complementary. The difference lies in the regulator's communication technology. Rather than observing the regulator's assessment directly, investors receive only a coarse public summary of it.

The regulator observes a continuous noisy signal

$$y = \theta + \varepsilon_R, \qquad \varepsilon_R \sim N\left(0, \sigma_R^2\right),$$

where ε_R is independent of the fundamental and the private signal disturbances. Since a higher value of θ represents greater financial weakness, y can be interpreted as the

regulator's assessment of the institution's underlying vulnerability.

Rather than disclosing the realization of this assessment, the regulator communicates whether it lies above or below a cutoff d. The cutoff can be interpreted as a regulatory benchmark separating relatively weak from relatively strong assessments of the institution's financial condition. The public message is therefore

$$m = \begin{cases} 1, & y \geq d, \\ 0, & y < d. \end{cases} \tag{25}$$

Thus, investors do not observe the exact value of the regulator's signal. Instead, they learn only which region of the signal space contains its realization. When $m = 1$, the regulator's assessment lies above the benchmark and therefore indicates relatively greater financial weakness. When $m = 0$, the assessment lies below the benchmark and indicates relatively lower financial weakness. Accordingly, the former tends to make withdrawal more attractive, whereas the latter tends to make continued investment more attractive.

For later use, it is convenient to represent the two possible messages using a single sign variable:

$$z = \begin{cases} 1, & m = 1, \\ -1, & m = 0. \end{cases}$$

Conditional on the fundamental, the probability of the observed message can then be written compactly as

$$\Pr(m \mid \theta) = \Phi\left(z\frac{\theta - d}{\sigma_R}\right).$$

The message therefore conveys information about the fundamental even though the regulator's underlying signal is not observed directly. The important distinction, however, is that the regulator's continuous assessment has been compressed into a binary communication. In the benchmark, conditioning on a continuous Gaussian public signal preserves the Gaussian form of posterior beliefs. Here, investors condition instead on the event that the regulator's signal lies above or below the cutoff. This truncation of the underlying Gaussian information generates a non-Gaussian posterior for the fundamental. The following subsection uses an Extended Skew-Normal representation to characterize these posterior beliefs and study the equilibrium implications of coarse regulatory communication.

3.2 The Posterior Distribution

Conditional on her private signal x_i, agent i's posterior is Gaussian. Using the precision notation introduced in the benchmark, let

$$\tau_0 = \frac{1}{\sigma_\theta^2}, \qquad \tau_x = \frac{1}{\sigma_x^2}. \tag{26}$$

Then

$$\theta \mid x_i \sim N(\mu_i, \sigma_i^2), \tag{27}$$

where

$$\sigma_i^2 = \frac{1}{\tau_0 + \tau_x}, \qquad \mu_i = \frac{\tau_0 \bar{\theta} + \tau_x x_i}{\tau_0 + \tau_x}. \tag{28}$$

The posterior variance σ_i^2 is common across agents, while the posterior mean μ_i depends on the realization of agent i's private signal.

Agent i subsequently observes the regulator's binary message m. Combining the Gaussian posterior based on x_i with the message likelihood derived above gives

$$p(\theta \mid x_i, m) \propto \frac{1}{\sigma_i} \phi\left(\frac{\theta - \mu_i}{\sigma_i}\right) \Phi\left(z \frac{\theta - d}{\sigma_R}\right). \tag{29}$$

Relative to the Gaussian posterior based on the private signal alone, the regulator's message therefore introduces a message-dependent selection term. The resulting posterior is no longer Gaussian.

This posterior belongs to the Extended Skew-Normal (ESN) family. The ESN family extends the Gaussian distribution by multiplying a normal density by a normal CDF term, and arises naturally when a Gaussian variable is conditioned on a Gaussian selection event. This is precisely the structure generated here: agents begin with a Gaussian posterior based on their private signal and then condition on the event that the regulator's noisy assessment lies on one side of the cutoff d. The binary message therefore introduces asymmetry into posterior beliefs. In the present setting,

$$\theta \mid x_i, m \sim ESN(\mu_i, \sigma_i, \alpha, \tau), \tag{30}$$

where

$$\alpha = \frac{z\sigma_i}{\sigma_R}, \qquad \tau = \frac{z(\mu_i - d)}{\sqrt{\sigma_R^2 + \sigma_i^2}}, \qquad \delta = \frac{z\sigma_i}{\sqrt{\sigma_R^2 + \sigma_i^2}}. \tag{31}$$

Here, α governs the asymmetry induced by the message, while τ captures the location of the Gaussian posterior relative to the regulatory cutoff. The parameter δ, which is a transformation of α, is convenient for expressing the moments of the posterior. The derivation of the ESN representation is provided in Appendix B.1.

It is convenient to define the inverse Mills ratio

$$\lambda(\tau) \equiv \frac{\phi(\tau)}{\Phi(\tau)}. \tag{32}$$

The posterior mean following the regulator's message is then

$$E[\theta \mid x_i, m] = \mu_i + \sigma_i \delta \lambda(\tau). \tag{33}$$

Denoting this posterior mean by μ_i^E,

$$\mu_i^E = \mu_i + \sigma_i \delta \lambda(\tau). \tag{34}$$

Thus, the regulator's message shifts the Gaussian posterior mean μ_i through the ESN correction term $\sigma_i \delta \lambda(\tau)$.

3.3 Coordination Incentives

We now turn to the coordination problem. Suppose that the other agents follow a symmetric cutoff strategy under which an agent keeps her funds invested if and only if her private signal is below a conjectured threshold t. We establish below that the best response is indeed monotone in the private signal, thereby justifying this threshold representation.

Given agent i's information (x_i, m), let $V(x_i, t, m)$ denote her expected payoff from keeping her funds invested when the other agents follow this conjectured strategy. Since agent i contributes $1/n$ to the fraction of agents who keep their funds invested, while each of the remaining $n-1$ agents does so when their private signal is below t, we have

$$V(x_i, t, m) = \frac{1}{n} + \frac{n-1}{n} \Pr(x_j < t \mid x_i, m) - \mathbb{E}[\theta \mid x_i, m]. \tag{35}$$

Using the conditional distribution of x_j given (x_i, m), the probability that another agent keeps her funds invested can be written as

$$\Pr(x_j < t \mid x_i, m) = \int \Phi\left(\frac{t-\theta}{\sigma_x}\right) p(\theta \mid x_i, m)\, d\theta. \tag{36}$$

Hence,

$$V(x_i, t, m) = \frac{1}{n} + \frac{n-1}{n} \int \Phi\left(\frac{t-\theta}{\sigma_x}\right) p(\theta \mid x_i, m)\, d\theta - \mu_i^E. \tag{37}$$

The first term captures agent i's own contribution to the fraction of agents who remain invested, the second captures the expected contribution of the other agents, and the final term captures the expected financial weakness conditional on the agent's information.

The threshold representation requires the best response to be monotone in the private

signal. We therefore seek to establish

$$\frac{\partial V(x_i, t, m)}{\partial x_i} < 0. \tag{38}$$

A higher private signal indicates greater financial weakness and therefore directly reduces the attractiveness of keeping funds invested. At the same time, it changes the agent's beliefs about whether other investors will remain invested. The result in the next section shows that the direct informational effect dominates this strategic effect, so that the best response is indeed characterized by a cutoff in x_i.

3.4 Monotonicity of the Best Response

We now establish that the best response is monotone in the private signal. Differentiating the value function in (37) with respect to x_i gives

$$\frac{\partial V(x_i, t, m)}{\partial x_i} = \frac{n-1}{n} \int \Phi\left(\frac{t-\theta}{\sigma_x}\right) \frac{\partial p(\theta \mid x_i, m)}{\partial x_i} \, d\theta - \frac{\partial \mu_i^E}{\partial x_i}. \tag{39}$$

The derivative captures two effects of a change in the private signal. First, a higher private signal changes the agent's posterior assessment of the fundamental. Second, through this change in beliefs, it affects her expectations about the actions of the other agents. We begin with the posterior mean. From (34),

$$\mu_i^E = \mu_i + \sigma_i \delta \lambda(\tau), \tag{40}$$

where $\lambda(\tau)$ is the inverse Mills ratio defined in (32). Since

$$\frac{\partial \mu_i}{\partial x_i} = \frac{\tau_x}{\tau_0 + \tau_x} > 0, \tag{41}$$

and the derivative of the inverse Mills ratio satisfies

$$-1 < \lambda'(\tau) < 0, \tag{42}$$

it follows that

$$\frac{\partial \mu_i^E}{\partial x_i} > 0. \tag{43}$$

Thus, even after conditioning on the regulator's coarse message, a higher private signal strictly raises the agent's posterior mean of the fundamental. The detailed derivation is provided in Appendix B.3.

The remaining term in (39) captures the effect of the private signal on the agent's beliefs about the actions of the other players. The technical derivation in Appendix B.3.1

allows this term to be expressed as a conditional covariance, yielding

$$\frac{\partial V(x_i, t, m)}{\partial x_i} = \frac{n-1}{n} \frac{\partial \mu_i}{\partial x_i} \frac{1}{\sigma_i^2} \operatorname{Cov}\left(\theta, \Pr(a_j = 1 \mid \theta) \mid x_i, m\right) - \frac{\partial \mu_i^E}{\partial x_i}. \tag{44}$$

To determine the sign of the first term, note that under the conjectured cutoff strategy,

$$\Pr(a_j = 1 \mid \theta) = \Phi\left(\frac{t - \theta}{\sigma_x}\right), \tag{45}$$

which is strictly decreasing in θ. Hence,

$$\operatorname{Cov}\left(\theta, \Pr(a_j = 1 \mid \theta) \mid x_i, m\right) < 0. \tag{46}$$

Since

$$\frac{\partial \mu_i}{\partial x_i} > 0, \qquad \sigma_i^2 > 0, \qquad \frac{\partial \mu_i^E}{\partial x_i} > 0,$$

both terms in (44) are strictly negative. We therefore obtain

$$\frac{\partial V(x_i, t, m)}{\partial x_i} < 0. \tag{47}$$

Lemma 1 (Monotonicity of the Best Response)**.** *For any conjectured threshold t and regulator message m, the payoff from keeping their funds $V(x_i, t, m)$ is strictly decreasing in the private signal x_i.*

Proof. See Appendix B.3 and Appendix B.3.1. The result has a simple economic interpretation. A higher private signal indicates greater financial weakness, which reduces the attractiveness of keeping funds invested through two reinforcing channels: it raises the agent's posterior assessment of the institution's weakness, while simultaneously lowering her assessment of the likelihood that other agents keep their funds invested. The best response is therefore characterized by a cutoff in the private signal.

3.5 Existence of the Threshold

Having established that the payoff from keeping funds invested is decreasing in the private signal, a threshold equilibrium can be characterized by the payoff of an agent whose private signal is exactly equal to the conjectured threshold. Define

$$G(t, m) \equiv V(t, t, m). \tag{48}$$

A threshold equilibrium for message m therefore satisfies

$$G(t, m) = 0. \tag{49}$$

Using the expression for the value function,

$$G(t,m) = \frac{1}{n} + \frac{n-1}{n}\Pr(x_j < t \mid x_i = t, m) - \mu_i^E(t,m). \tag{50}$$

The coordination component is bounded between zero and one. Hence,

$$\frac{1}{n} - \mu_i^E(t,m) \leq G(t,m) \leq 1 - \mu_i^E(t,m). \tag{51}$$

It therefore suffices to characterize the limiting behavior of the posterior mean. Since

$$\mu_i(t) = \frac{\tau_0\bar{\theta} + \tau_x t}{\tau_0 + \tau_x}, \tag{52}$$

we have

$$\lim_{t\to-\infty} \mu_i(t) = -\infty, \qquad \lim_{t\to+\infty} \mu_i(t) = +\infty. \tag{53}$$

The posterior mean following the regulator's message is

$$\mu_i^E(t,m) = \mu_i(t) + \sigma_i \delta \lambda(\tau), \tag{54}$$

where

$$\delta = \frac{z\sigma_i}{\sqrt{\sigma_R^2 + \sigma_i^2}}, \qquad \tau = \frac{z(\mu_i(t) - d)}{\sqrt{\sigma_R^2 + \sigma_i^2}}. \tag{55}$$

Although the regulatory message changes the posterior through the ESN correction, it does not alter the tail behavior of the posterior mean. In particular,

$$\lim_{t\to-\infty} \mu_i^E(t,m) = -\infty, \qquad \lim_{t\to+\infty} \mu_i^E(t,m) = +\infty. \tag{56}$$

The detailed tail argument is provided in Appendix B.4.

Combining (51) with (56) gives

$$\lim_{t\to-\infty} G(t,m) = +\infty, \qquad \lim_{t\to+\infty} G(t,m) = -\infty. \tag{57}$$

Finally, $G(t,m)$ is continuous in t, since both the conditional coordination probability and the posterior mean are continuous in the threshold. Hence, by the intermediate value theorem, there exists a finite $t^*(m)$ satisfying

$$G(t^*(m), m) = 0. \tag{58}$$

Proposition 2 (Existence of a Threshold Equilibrium)**.** *For each regulator message $m \in \{0,1\}$, there exists at least one finite threshold equilibrium $t^*(m)$.*

The result establishes existence but does not rule out multiple threshold equilibria. We therefore turn next to the conditions under which the threshold is unique.

3.6 Uniqueness and Equilibrium Characterization

3.6.1 Uniqueness

Having established existence, we now turn to uniqueness. An equilibrium threshold is a solution to

$$G(t, m) = V(t, t, m) = 0. \tag{59}$$

By the chain rule,

$$\frac{dG(t,m)}{dt} = \frac{\partial V(t,t,m)}{\partial x_i} + \frac{\partial V(t,t,m)}{\partial t}. \tag{60}$$

From the monotonicity result established above,

$$\frac{\partial V}{\partial x_i} < 0.$$

On the other hand, increasing the conjectured threshold makes it more likely that other agents keep their funds invested. Differentiating the coordination probability gives

$$\frac{\partial V(x_i,t,m)}{\partial t} = \frac{n-1}{n\sigma_x} \int \phi\left(\frac{t-\theta}{\sigma_x}\right) p(\theta \mid x_i, m)\, d\theta > 0. \tag{61}$$

Thus, the two effects enter (60) with opposite signs: a higher private signal, which indicates greater financial weakness, reduces the incentive to keep funds invested, while a higher conjectured threshold increases the expected coordination benefit from doing so. Consequently,

$$\frac{dG(t,m)}{dt} < 0 \tag{62}$$

whenever

$$\left|\frac{\partial V(t,t,m)}{\partial x_i}\right| > \frac{\partial V(t,t,m)}{\partial t}. \tag{63}$$

Under this condition, $G(t, m)$ is strictly decreasing in t and can therefore cross zero at most once. Combined with the existence result, this establishes uniqueness.

To obtain a tractable sufficient condition, we bound the derivative with respect to the conjectured threshold. The detailed evaluation of (61), using the product of two Gaussian densities and the Extended Skew-Normal representation of the posterior, is provided in Appendix B.5. It yields

$$\frac{\partial V(x_i,t,m)}{\partial t} \leq \frac{n-1}{n} \frac{1}{\sqrt{2\pi(\sigma_x^2+\sigma_i^2)}\Phi(\tau)}. \tag{64}$$

Hence, a sufficient condition for uniqueness is

$$\left|\frac{\partial V(t,t,m)}{\partial x_i}\right| \geq \frac{n-1}{n} \frac{1}{\sqrt{2\pi(\sigma_x^2+\sigma_i^2)}\Phi(\tau(t,m))} \tag{65}$$

for all t.

We can further obtain a lower bound on the magnitude of the response to the private signal. From (44),

$$\frac{\partial V(x_i,t,m)}{\partial x_i} = \frac{n-1}{n}\frac{\partial \mu_i}{\partial x_i}\frac{1}{\sigma_i^2}\operatorname{Cov}\left(\theta, \Pr(a_j = 1 \mid \theta)|x_i, m\right) - \frac{\partial \mu_i^E}{\partial x_i}. \tag{66}$$

The covariance term is strictly negative, while

$$\frac{\partial \mu_i^E}{\partial x_i} > 0.$$

Therefore,

$$\left|\frac{\partial V(x_i,t,m)}{\partial x_i}\right| > \frac{\partial \mu_i^E}{\partial x_i}. \tag{67}$$

It follows that a stronger, purely informational sufficient condition for uniqueness is

$$\frac{\partial \mu_i^E}{\partial x_i} \geq \frac{n-1}{n}\frac{1}{\sqrt{2\pi(\sigma_x^2+\sigma_i^2)}\Phi(\tau(t,m))} \tag{68}$$

for all t.

The condition has a natural interpretation. The left-hand side measures how strongly the agent's posterior assessment of the institution's financial weakness responds to her own private information. The right-hand side bounds the strategic effect generated by a change in the conjectured threshold. Uniqueness is therefore guaranteed when the informational response to the agent's own signal is sufficiently strong relative to the coordination response to the conjectured threshold.

Proposition 3 (Uniqueness of the Threshold Equilibrium)**.** *For each regulator message $m \in \{0,1\}$, suppose that* (68) *holds for all t. Then the threshold equilibrium is unique.*

3.6.2 Equilibrium Threshold

The equilibrium threshold is characterized by the indifference condition of an agent whose private signal is exactly equal to the conjectured threshold. Thus, for each message m, the equilibrium threshold t_m^* satisfies

$$V(t_m^*, t_m^*, m) = 0. \tag{69}$$

Substituting the posterior distribution and the coordination probability derived above gives an implicit characterization of the equilibrium threshold. Define

$$S_x \equiv \sqrt{\sigma_x^2+\sigma_i^2}, \qquad S_R \equiv \sqrt{\sigma_R^2+\sigma_i^2}, \tag{70}$$

and

$$a(t) \equiv \frac{t - \mu_i(t)}{S_x}, \qquad \tau(t, m) \equiv \frac{z(\mu_i(t) - d)}{S_R}, \tag{71}$$

where

$$\rho \equiv \frac{z\sigma_i^2}{S_x S_R}. \tag{72}$$

The posterior mean at the threshold is

$$\mu_i^E(t, m) = \mu_i(t) + \frac{z\sigma_i^2}{S_R} \frac{\phi\left(\tau(t, m)\right)}{\Phi\left(\tau(t, m)\right)}. \tag{73}$$

The probability that another agent keeps her funds invested can be expressed in terms of the bivariate standard normal distribution as

$$\Pr(x_j < t \mid x_i = t, m) = \frac{\Phi_2\left(a(t), \tau(t, m); -\rho\right)}{\Phi\left(\tau(t, m)\right)}. \tag{74}$$

Hence, the equilibrium threshold is implicitly characterized by

$$\begin{aligned} 0 = \frac{1}{n} + \frac{n-1}{n} \frac{\Phi_2\left(a(t_m^*), \tau(t_m^*, m); -\rho\right)}{\Phi\left(\tau(t_m^*, m)\right)} \\ - \mu_i(t_m^*) - \frac{z\sigma_i^2}{S_R} \frac{\phi\left(\tau(t_m^*, m)\right)}{\Phi\left(\tau(t_m^*, m)\right)}. \end{aligned} \tag{75}$$

where

$$\mu_i(t) = \frac{\tau_0 \bar{\theta} + \tau_x t}{\tau_0 + \tau_x}. \tag{76}$$

In general, (75) does not admit a closed-form solution. Under the sufficient condition for uniqueness established above, however, it uniquely characterizes the equilibrium threshold for each regulator message m.

3.7 Limiting Cases

The implications of coarse regulatory communication become particularly transparent in two limiting cases. When $\sigma_R \to \infty$, the regulator's assessment becomes uninformative, and the model converges to the benchmark without meaningful regulatory communication. When $\sigma_R \to 0$, by contrast, the regulator observes the institution's underlying condition arbitrarily precisely, but its binary message continues to reveal only on which side of the cutoff d that condition lies. These two limits therefore highlight the distinction between the precision of the regulator's information and the amount of information conveyed through its communication. They also provide useful benchmarks for the uniqueness condition and the equilibrium threshold.

3.7.1 Uninformative Regulator: $\sigma_R \to \infty$

As $\sigma_R \to \infty$, the regulator's signal becomes uninformative. In particular,

$$\alpha \to 0, \qquad \tau \to 0, \qquad \rho \to 0,$$

and the ESN correction to the posterior mean vanishes:

$$\mu_i^E(t, m) \to \mu_i(t).$$

The sufficient condition for uniqueness therefore converges to

$$\frac{\sigma_i^2}{\sigma_x^2} \geq \frac{n-1}{n}\sqrt{\frac{2}{\pi(\sigma_x^2 + \sigma_i^2)}}. \tag{77}$$

At the same time, the equilibrium threshold equation converges to

$$\frac{1}{n} + \frac{n-1}{n}\Phi\left(\frac{t - \mu_i(t)}{\sqrt{\sigma_x^2 + \sigma_i^2}}\right) - \mu_i(t) = 0. \tag{78}$$

Thus, in the limit, the regulator's message carries no information beyond that already contained in agents' private signals. The equilibrium threshold is independent of both m and d, and the model converges to the benchmark in which agents rely exclusively on their private information.

3.7.2 Perfectly Informed but Coarse Regulator: $\sigma_R \to 0$

Consider now the opposite limit, $\sigma_R \to 0$. The regulator observes the fundamental arbitrarily precisely, so that

$$y \to \theta.$$

Because communication remains binary, however, the message converges to

$$m = \begin{cases} 1, & \theta \geq d, \\ 0, & \theta < d. \end{cases}$$

Thus, even though the regulator is perfectly informed, agents learn only which side of the cutoff d the fundamental lies on. They do not observe the precise realization of θ.

In this limit,

$$\tau(t) \to \frac{z(\mu_i(t) - d)}{\sigma_i},$$

and the posterior mean becomes

$$\mu_i^E(t,m) \to \mu_i(t) + z\sigma_i \frac{\phi\left(\frac{z(\mu_i(t)-d)}{\sigma_i}\right)}{\Phi\left(\frac{z(\mu_i(t)-d)}{\sigma_i}\right)}. \tag{79}$$

The posterior is therefore a truncated normal distribution: conditional on $m = 1$, the posterior is truncated below at d, while conditional on $m = 0$, it is truncated above at d.

The sufficient condition for uniqueness converges to

$$\frac{\sigma_i^2}{\sigma_x^2}\left[1 - \lambda(\tau)\left(\tau + \lambda(\tau)\right)\right] \geq \frac{n-1}{n}\frac{1}{\sqrt{2\pi(\sigma_x^2+\sigma_i^2)}\Phi(\tau)}, \tag{80}$$

where

$$\tau = \frac{z(\mu_i(t) - d)}{\sigma_i}.$$

The equilibrium threshold is correspondingly characterized by the limiting version of the threshold equation in (75).

This case illustrates the central distinction underlying the model. The regulator's information can become arbitrarily precise without its communication becoming fully revealing. Even in the limit of a perfectly informed regulator, the binary message only partitions the state space at the cutoff d. Residual uncertainty about θ therefore remains, so agents' private signals continue to play an informational role. In contrast to a fully revealing public signal, coarse communication need not eliminate the strategic uncertainty underlying the coordination problem and, under the sufficient condition above, can therefore remain compatible with a unique equilibrium.

Having established that equilibrium existence and uniqueness can be preserved under sufficiently coarse communication, we now examine how the communication rule itself affects the equilibrium threshold.

3.8 Comparative Statics

We finally study how the regulator's communication cutoff d affects the equilibrium threshold. The equilibrium threshold t_m^* satisfies

$$G(t_m^*, d; m) = 0, \tag{81}$$

where

$$G(t,d;m) = \frac{1}{n} + \frac{n-1}{n}F(t,d;m) - \mu_i^E(t,d;m). \tag{82}$$

For a fixed threshold t, the cutoff d affects the posterior only through

$$\tau(t,d) = \frac{z(\mu_i(t) - d)}{\sqrt{\sigma_R^2 + \sigma_i^2}}, \tag{83}$$

so that

$$\frac{\partial \tau}{\partial d} = -\frac{z}{\sqrt{\sigma_R^2 + \sigma_i^2}}. \tag{84}$$

Consider first the coordination probability

$$F(t,d;m) = \Pr(x_j < t \mid x_i = t, m). \tag{85}$$

Conditional on $x_i = t$, the regulator's signal y and the other agent's private signal x_j are positively correlated:

$$\mathrm{Cov}(x_j, y \mid x_i = t) = \sigma_i^2 > 0. \tag{86}$$

An increase in d therefore changes the conditioning event in a way that shifts the conditional distribution of x_j upward, for either realized message. Hence, the probability that another agent's signal lies below the threshold falls:

$$\frac{\partial F(t,d;m)}{\partial d} < 0, \qquad m \in \{0,1\}. \tag{87}$$

The formal derivative is provided in Appendix B.10.

Next consider the posterior mean,

$$\mu_i^E(t,d;m) = \mu_i(t) + \frac{z\sigma_i^2}{\sqrt{\sigma_R^2 + \sigma_i^2}}\lambda(\tau), \tag{88}$$

where

$$\lambda(\tau) = \frac{\phi(\tau)}{\Phi(\tau)}. \tag{89}$$

Using

$$\lambda'(\tau) = -\lambda(\tau)\left[\tau + \lambda(\tau)\right] \tag{90}$$

and (84), we obtain

$$\frac{\partial \mu_i^E}{\partial d} = \frac{\sigma_i^2}{\sigma_R^2 + \sigma_i^2}\lambda(\tau)\left[\tau + \lambda(\tau)\right] > 0. \tag{91}$$

Thus, holding the realized message fixed, increasing the regulatory cutoff raises the posterior assessment of the institution's financial weakness. The interpretation is slightly different across messages. When $m = 1$, the message indicates that the regulator's signal has exceeded the cutoff, so a higher cutoff means that the same message is associated with a more extreme assessment of financial weakness. When $m = 0$, raising the cutoff makes the event $y < d$ less restrictive and allows relatively weaker realizations to be consistent

with the same message. In both cases, therefore, a higher cutoff raises the conditional expectation of θ.

Since higher values of θ correspond to greater financial weakness, this makes keeping funds invested less attractive. Moreover, because (87) implies that a higher cutoff lowers the probability that another investor's signal lies below the threshold, it also changes the expected coordination benefit associated with keeping funds invested.

Differentiating the equilibrium condition with respect to d and applying the implicit function theorem gives

$$\frac{dt_m^*}{dd} = -\frac{G_d(t_m^*, d; m)}{G_t(t_m^*, d; m)}. \tag{92}$$

From the expression for G,

$$G_d = \frac{n-1}{n}\frac{\partial F}{\partial d} - \frac{\partial \mu_i^E}{\partial d}. \tag{93}$$

By (87) and (91),

$$G_d < 0. \tag{94}$$

Under the sufficient condition for uniqueness established above,

$$G_t(t_m^*, d; m) < 0. \tag{95}$$

Consequently,

$$\frac{dt_m^*}{dd} < 0. \tag{96}$$

Proposition 4 (Comparative Statics with Respect to the Regulatory Cutoff)**.** *Holding the received regulatory message fixed, an increase in the regulator's communication cutoff d strictly lowers the equilibrium threshold:*

$$\frac{dt_m^*}{dd} < 0, \qquad m \in \{0, 1\}. \tag{97}$$

The economic intuition follows from the two effects above. Holding the realized message fixed, a higher regulatory cutoff makes that message consistent with greater expected financial weakness and therefore reduces the attractiveness of keeping funds invested. At the same time, it changes beliefs about the behavior of other investors through the coordination term. Both effects reduce the payoff from keeping funds invested at a given private signal. The equilibrium cutoff must therefore fall in order to restore indifference: investors require a more favorable private signal before they are willing to keep their funds invested.

The cutoff also affects the ex ante frequency with which each message is received. For example,

$$\Pr(m = 1 \mid x_i = t) = \Phi\left(\frac{\mu_i(t) - d}{\sqrt{\sigma_R^2 + \sigma_i^2}}\right), \tag{98}$$

which is decreasing in d. Thus, the regulatory cutoff affects both the frequency with which each message is observed and behavior conditional on the message.

4 Information Transmission and Network Structure

The benchmark model abstracts from decentralized information transmission between investors. Investors receive private signals about the institution's financial condition alongside a common public signal from the regulator. In many financial settings, however, information may also spread through social, professional, or financial connections. The failure of Silicon Valley Bank provides a salient example. Cookson et al. (2026) document how communication on social media contributed to the transmission of concerns among depositors, highlighting the potential for decentralized networks to shape collective withdrawal decisions. We therefore extend the model to examine how regulatory communication interacts with private information distributed through a network.

4.1 The Network and Information Structure

The underlying coordination problem remains unchanged. Investors decide whether to keep their funds invested in the institution or withdraw them, with their decisions depending both on their assessment of the institution's underlying condition and on their expectations about the actions of other investors.

The fundamental θ and the private signals $\{x_i\}_{i\in\mathcal{I}}$ are as defined in the benchmark. The new element is the structure through which investors observe private signals. Investors are connected through an undirected network

$$\mathcal{G} = (\mathcal{I}, E),$$

where

$$\mathcal{I} = \{1, \ldots, n\}$$

is the set of investors and $(i, j) \in E$ indicates that investors i and j are directly connected. A link gives the two connected investors access to each other's private signals. The network therefore determines how the otherwise dispersed private information is distributed across investors.

Let

$$\mathcal{N}_i = \{i\} \cup \{j \in \mathcal{I} : (i, j) \in E\} \tag{99}$$

denote the set of private signals observed by investor i, including her own signal and those of her neighbours. Let

$$k_i = |\mathcal{N}_i| \tag{100}$$

denote the number of private signals available to her. Thus, different network positions generally imply different, and potentially overlapping, information sets.

All investors also observe the common public signal provided by the regulator, as in the benchmark:

$$y = \theta + \varepsilon_R, \tag{101}$$

where

$$\varepsilon_R \sim N(0, \sigma_R^2), \tag{102}$$

independently of the fundamental and of all private-signal disturbances. We again denote the precision of the regulator's signal by

$$\gamma = \frac{1}{\sigma_R^2}. \tag{103}$$

Investor i's information set is therefore

$$\mathcal{I}_i = \left(\{x_j\}_{j \in \mathcal{N}_i}, y\right). \tag{104}$$

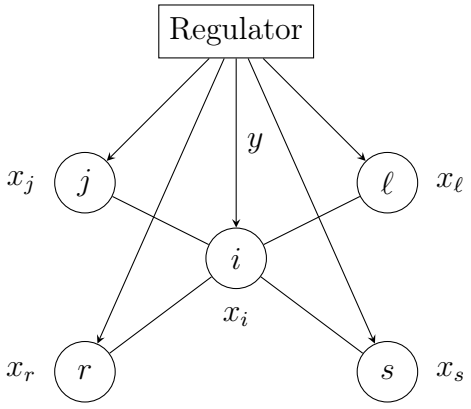


Figure 1: Information available to an investor through the network and the regulator. Investor i observes the private signals indexed by $\mathcal{N}_i$, including her own signal, and the common public signal y.

The network affects the coordination problem through two related channels. First, additional connections expand the set of private signals available to an investor and therefore affect her assessment of the institution's underlying condition. Second, because different investors may observe different and overlapping sets of signals, network structure also shapes what each investor can infer about the information available to others. It therefore affects not only first-order beliefs about financial weakness, but also the higher-order beliefs that are central to withdrawal decisions and coordination.

We focus on threshold equilibria. Each investor i keeps her funds invested whenever her posterior assessment $\hat{\theta}_i$ lies below a cutoff $t_i(y)$. A profile of thresholds $\{t_i(y)\}_{i \in \mathcal{I}}$ constitutes an equilibrium if each investor's cutoff is optimal given the threshold strategies

of all other investors.

The analysis below examines how these decentralized information channels interact with the common signal provided by the regulator. In particular, we study how regulatory precision affects both the role of network information in belief formation and investors' uncertainty about one another's beliefs.

4.2 Beliefs and Higher-Order Beliefs

We now characterize how the network and the regulator's public signal shape investors' beliefs about the institution's underlying financial condition and their beliefs about what other investors know. The network determines which private assessments of financial weakness are available to each investor, while the regulator provides a common source of information observed by all.

4.2.1 Posterior Beliefs

Conditional on the underlying fundamental θ, all signals are independent. Investor i therefore combines k_i private signals, each with precision β, with the regulator's public signal, which has precision γ. Her posterior precision is consequently

$$D_i = \alpha + \beta k_i + \gamma. \tag{105}$$

It follows that investor i's posterior mean is

$$\hat{\theta}_i \equiv \mathbb{E}_i[\theta] = \frac{\alpha\bar{\theta} + \beta \sum_{\ell \in \mathcal{N}_i} x_\ell + \gamma y}{D_i}. \tag{106}$$

The corresponding posterior variance is

$$\mathrm{Var}_i(\theta) = \frac{1}{D_i}. \tag{107}$$

Thus, both additional network links and a more precise public signal reduce an investor's uncertainty about the institution's underlying condition. The two sources of information, however, differ in how they are distributed. Network links expand the set of signals available to particular investors and can therefore generate heterogeneous posterior beliefs, whereas the regulator's signal enters every investor's information set.

This distinction becomes important for coordination because investors must also form expectations about how others assess the institution and how they are likely to act. We therefore turn next to higher-order beliefs, where the network structure determines what an investor can infer about the information and beliefs of other investors.

4.2.2 Beliefs about Other Investors' Posterior Beliefs

We next consider investor i's beliefs about investor j's assessment of the institution's underlying financial condition. While investor i can form a posterior belief about θ using her own information, coordination also requires her to assess what other investors are likely to believe. In particular, investor j may observe private signals that are not contained in investor i's information set.

Investor j's posterior mean is

$$\hat{\theta}_j = \frac{\alpha\bar{\theta} + \beta \sum_{\ell \in \mathcal{N}_j} x_\ell + \gamma y}{\alpha + \beta k_j + \gamma}. \tag{108}$$

From the perspective of investor i, some of the private signals entering $\hat{\theta}_j$ are also observed by i, while others are not. Let

$$\mathcal{C}_{ij} = \mathcal{N}_i \cap \mathcal{N}_j \tag{109}$$

denote the set of private signals observed by both investors. The remaining signals in investor j's information set are contained in $\mathcal{N}_j \setminus \mathcal{N}_i$. Conditional on $\mathcal{I}_i$, investor i observes the realizations in $\mathcal{C}_{ij}$ but must form expectations about the remaining signals.

Since, for any signal x_ℓ not observed by i where $\ell \in \mathcal{N}_j \setminus \mathcal{N}_i$,

$$\mathbb{E}_i[x_\ell] = \hat{\theta}_i,$$

investor i's expectation of investor j's posterior belief is

$$\mathbb{E}_i[\hat{\theta}_j] = \frac{1}{\alpha + \beta k_j + \gamma} \left[\alpha\bar{\theta} + \beta \sum_{\ell \in \mathcal{C}_{ij}} x_\ell + \beta \left| \mathcal{N}_j \setminus \mathcal{N}_i \right| \hat{\theta}_i + \gamma y \right]. \tag{110}$$

The expression separates the information underlying investor j's posterior into signals that investor i already observes and signals that remain unobserved. The extent of overlap between their informational neighbourhoods therefore determines how precisely investor i can infer investor j's assessment of the institution.

4.2.3 Uncertainty about Other Investors' Beliefs

We now characterize investor i's residual uncertainty about investor j's posterior belief. Conditional on $\mathcal{I}_i$, uncertainty about $\hat{\theta}_j$ arises only from the private signals that enter investor j's information set but are not observed by investor i. Let

$$m_{ij} = \left| \mathcal{N}_j \setminus \mathcal{N}_i \right| \tag{111}$$

denote the number of such signals.

Signals observed by both investors are known conditional on $\mathcal{I}_i$ and therefore do not contribute to investor i's uncertainty about $\hat{\theta}_j$. It follows that

$$\operatorname{Var}_i\left(\hat{\theta}_j \mid \mathcal{I}_i\right) = \frac{\beta^2}{D_j^2} \operatorname{Var}_i\left(\sum_{\ell \in \mathcal{N}_j \setminus \mathcal{N}_i} x_\ell \middle| \mathcal{I}_i\right). \tag{112}$$

The unobserved signals are not conditionally independent from investor i's perspective because they share the common fundamental θ. For any two distinct unobserved signals $\ell \neq r$,

$$\operatorname{Cov}_i\left(x_\ell, x_r \mid \mathcal{I}_i\right) = \operatorname{Var}_i\left(\theta \mid \mathcal{I}_i\right) = \frac{1}{D_i}. \tag{113}$$

Moreover, for each unobserved signal,

$$\operatorname{Var}_i\left(x_\ell \mid \mathcal{I}_i\right) = \operatorname{Var}_i\left(\theta \mid \mathcal{I}_i\right) + \operatorname{Var}\left(\varepsilon_\ell\right) = \frac{1}{D_i} + \frac{1}{\beta}. \tag{114}$$

Therefore,

$$\begin{aligned} &\operatorname{Var}_i\left(\sum_{\ell \in \mathcal{N}_j \setminus \mathcal{N}_i} x_\ell \middle| \mathcal{I}_i\right) \\ &= m_{ij}\left(\frac{1}{D_i} + \frac{1}{\beta}\right) + m_{ij}(m_{ij} - 1)\frac{1}{D_i} \\ &= \frac{m_{ij}^2}{D_i} + \frac{m_{ij}}{\beta}. \end{aligned} \tag{115}$$

Substituting into (112) yields

$$\operatorname{Var}_i\left(\hat{\theta}_j \mid \mathcal{I}_i\right) = \frac{\beta^2}{D_j^2}\left(\frac{m_{ij}^2}{D_i} + \frac{m_{ij}}{\beta}\right). \tag{116}$$

Equivalently,

$$\operatorname{Var}_i\left(\hat{\theta}_j \mid \mathcal{I}_i\right) = \frac{\beta m_{ij}}{D_j^2}\left(1 + \frac{\beta m_{ij}}{D_i}\right). \tag{117}$$

The expression shows that higher-order uncertainty is governed by the information underlying investor j's posterior that remains unobserved by investor i. As m_{ij} increases, investor i becomes less certain about how j assesses the institution's financial weakness. By contrast, the common public signal enters both investors' information sets and reduces this residual uncertainty through its effect on their posterior precisions.

The compact representation will be useful in the comparative statics below.

4.3 Equilibrium Thresholds and Uniqueness

We now turn to the strategic component of the model. As in the benchmark analysis, investors choose whether to withdraw their funds from the financial institution or keep them invested. The payoff from keeping funds invested depends both on the institution's underlying financial condition and on the decisions of other investors. The network structure enters the coordination problem through the information available to investors and, consequently, through their beliefs about whether others will withdraw or continue providing funding.

To characterize such an equilibrium, consider investor i's best response to the threshold strategies followed by the other investors. In particular, investor j keeps her funds invested whenever

$$\hat{\theta}_j \leq t_j(y).$$

Given investor i's information set I_i, the expected payoff from keeping her funds invested is

$$V_i(t_{-i} \mid \mathcal{I}_i) = \frac{1}{n} + \frac{1}{n} \sum_{j \neq i} \Pr_i \left(\hat{\theta}_j \leq t_j(y) \mid \mathcal{I}_i \right) - \hat{\theta}_i. \tag{118}$$

The expected payoff is decreasing in investor i's posterior assessment of the fundamental. Her best response is therefore characterized by a cutoff $t_i(y)$, such that

$$a_i = \begin{cases} 1, & \text{if } \hat{\theta}_i < t_i(y), \\ 0, & \text{if } \hat{\theta}_i > t_i(y), \end{cases} \tag{119}$$

where $a_i = 1$ denotes keeping funds invested.

At the threshold, investor i must be indifferent between keeping her funds invested and withdrawing them. Thus,

$$V_i(t_{-i} \mid \mathcal{I}_i) = 0 \qquad \text{whenever} \qquad \hat{\theta}_i = t_i(y). \tag{120}$$

From the analysis above, investor i's uncertainty about investor j's posterior mean is given by

$$\hat{\theta}_j \mid \mathcal{I}_i \sim \mathcal{N}\left(\mathbb{E}_i[\hat{\theta}_j], V_{ij} \right), \tag{121}$$

where

$$V_{ij} = \mathrm{Var}_i \left(\hat{\theta}_j \mid \mathcal{I}_i \right).$$

It follows that

$$\Pr_i \left(\hat{\theta}_j \leq t_j(y) \mid \mathcal{I}_i \right) = \Phi \left(\frac{t_j(y) - \mathbb{E}_i[\hat{\theta}_j \mid \mathcal{I}_i]}{\sqrt{V_{ij}}} \right). \tag{122}$$

Substituting (122) into the indifference condition (120) yields investor i's threshold

equation:

$$t_i(y) = \frac{1}{n} + \frac{1}{n} \sum_{j \neq i} \Phi \left(\frac{t_j(y) - \mathbb{E}_i[\hat{\theta}_j \mid \mathcal{I}_i]}{\sqrt{V_{ij}}} \right). \tag{123}$$

Equation (123) defines a system of interdependent threshold equations. To study whether this system admits a unique solution, let

$$BR = (BR_1, \ldots, BR_n)$$

denote the corresponding best-response mapping, with

$$BR_i(t_{-i}; \mathcal{I}_i) = \frac{1}{n} + \frac{1}{n} \sum_{j \neq i} \Phi \left(\frac{t_j(y) - \mathbb{E}_i[\hat{\theta}_j \mid \mathcal{I}_i]}{\sqrt{V_{ij}}} \right). \tag{124}$$

To study uniqueness, consider the sensitivity of investor i's best response with respect to the threshold of another investor j. Differentiating (124) gives

$$\frac{\partial BR_i}{\partial t_j} = \frac{1}{n\sqrt{V_{ij}}} \phi \left(\frac{t_j(y) - \mathbb{E}_i[\hat{\theta}_j \mid \mathcal{I}_i]}{\sqrt{V_{ij}}} \right), \tag{125}$$

where ϕ denotes the density of the standard normal distribution. Since

$$\phi(z) \leq \frac{1}{\sqrt{2\pi}} \qquad \text{for all } z,$$

we obtain the bound

$$\left| \frac{\partial BR_i}{\partial t_j} \right| \leq \frac{1}{n\sqrt{2\pi V_{ij}}}. \tag{126}$$

A sufficient condition for the Jacobian of the best-response mapping to have row sums strictly smaller than one is

$$2\pi V_{ij} > 1 \qquad \text{for all } i \neq j. \tag{127}$$

Under this condition,

$$\left| \frac{\partial BR_i}{\partial t_j} \right| < \frac{1}{n},$$

and hence the row sum of the Jacobian of the best-response mapping is strictly smaller than one:

$$\sum_{j \neq i} \left| \frac{\partial BR_i}{\partial t_j} \right| < \frac{n-1}{n} < 1.$$

Thus, the best-response operator is a contraction under the sup norm. By the Banach fixed-point theorem, the system of threshold equations admits a unique fixed point.

The uniqueness condition can now be expressed directly in terms of the network and

information structure. Using the expression derived above,

$$V_{ij} = \frac{\beta^2}{D_j^2}\left(\frac{m_{ij}}{\beta} + \frac{m_{ij}^2}{D_i}\right), D_i = \alpha + \beta k_i + \gamma, \qquad D_j = \alpha + \beta k_j + \gamma, \tag{128}$$

where m_{ij} captures the number of signals observed by investor j but not by investor i. Substituting (128) into (127) yields the following sufficient condition for uniqueness:

$$\frac{2\pi\beta^2}{[\alpha + \beta k_j + \gamma]^2}\left[\frac{m_{ij}}{\beta} + \frac{m_{ij}^2}{\alpha + \beta k_i + \gamma}\right] > 1. \tag{129}$$

This expression makes explicit the role of both network structure and regulatory communication in determining equilibrium uniqueness. The key object is the residual uncertainty that investor i faces about investor j's posterior belief. In a withdrawal environment, what matters for coordination is not only whether investors have information about the institution's underlying condition, but also whether they can infer how other investors interpret the information available to them and, therefore, whether they are likely to continue providing funding.

This residual uncertainty depends on the informational asymmetry generated by the network, summarized by m_{ij}, and on the amount of information available to each investor, summarized by k_i and k_j. The regulator's public signal affects this uncertainty through the common precision parameter γ.

The condition also highlights an important distinction between the amount of information circulating through the network and the informational differences across investors. When m_{ij} is small, investor i has little uncertainty about the additional information incorporated into investor j's posterior. Indeed,

$$m_{ij} \to 0 \qquad \Longrightarrow \qquad V_{ij} \to 0.$$

In this case, investor i can increasingly infer how investor j assesses the institution's financial condition and therefore faces less uncertainty about j's likely action. As V_{ij} becomes small, the upper bound on the sensitivity of best responses increases, so the sufficient contraction condition may fail. Conversely, informational differences generated by the network increase residual uncertainty about other investors' beliefs and can weaken the coordination forces that underlie self-fulfilling withdrawals.

The residual uncertainty also vanishes at the extremes of private-signal precision. As

$$\beta \to 0,$$

private signals become uninformative and the common sources of information dominate, implying that

$$V_{ij} \to 0.$$

At the other extreme, as

$$\beta \to \infty,$$

the private signals observed by investors become arbitrarily precise and

$$V_{ij} \to 0$$

again. Thus, residual uncertainty about other investors' posterior beliefs is highest, if at all, at an intermediate level of private-signal precision, suggesting that the sufficient condition for uniqueness is most likely to hold away from the two extremes.

Finally, the role of regulatory communication can be seen directly by setting $\gamma = 0$. In the absence of a public signal from the regulator, the sufficient condition reduces to

$$\frac{2\pi\beta^2}{[\alpha + \beta k_j]^2}\left[\frac{m_{ij}}{\beta} + \frac{m_{ij}^2}{\alpha + \beta k_i}\right] > 1. \tag{130}$$

Comparing (129) with (130) makes explicit how the common public signal enters the uniqueness condition through both investors' posterior precisions. The following section examines this role directly through comparative statics with respect to the precision of regulatory communication.

4.4 The Role of Network Structure

We now examine how the regulator's public signal affects the role of the network in investors' beliefs and strategic uncertainty. The public signal affects the model at two distinct levels. First, it changes the weight that investors place on the decentralized private information available through the network. Second, it reduces the residual uncertainty that investors face about the beliefs of other investors. We consider each channel in turn.

4.4.1 The Influence of Network Signals on Posterior Beliefs

Recall that investor i's posterior mean is given by

$$\hat{\theta}_i = \frac{\alpha\bar{\theta} + \beta\sum_{\ell\in\mathcal{N}_i} x_\ell + \gamma y}{D_i}. \tag{131}$$

Consider any private signal x_ℓ observed by investor i, where $\ell \in \{i\}\cup\mathcal{N}_i$. The marginal effect of this signal on investor i's posterior mean is

$$\frac{\partial\hat{\theta}_i}{\partial x_\ell} = \frac{\beta}{D_i}. \tag{132}$$

The precision of the regulator's public signal enters the denominator D_i directly. Dif-

ferentiating (132) with respect to γ therefore gives

$$\frac{\partial}{\partial \gamma}\left(\frac{\partial \hat{\theta}_i}{\partial x_\ell}\right) = \frac{\partial^2 \hat{\theta}_i}{\partial \gamma \, \partial x_\ell} = -\frac{\beta}{D_i^2} < 0. \tag{133}$$

Thus, an increase in the precision of the regulator's public signal reduces the marginal influence of every individual private signal observed through the network. As the public signal becomes more precise, investors place less weight on any particular decentralized source of information when forming their posterior beliefs.

This provides a first sense in which regulatory communication attenuates the role of the network in the coordination problem. The network continues to transmit decentralized information, but the marginal contribution of that information to investors' beliefs declines as the common public signal becomes more informative. This result extends the crowding-out effect of public information from the benchmark model to a network setting. In the benchmark, a more precise public signal reduces the role of investors' private information in belief formation. The same mechanism operates here: although private information is now distributed through a network, a more informative public signal reduces the marginal contribution of network-based information to investors' beliefs.

4.4.2 Regulatory Communication and Higher-Order Uncertainty

Regulatory communication also affects the role of the network through higher-order beliefs. The relevant object is the residual uncertainty V_{ij} that investor i faces about investor j's posterior belief. As shown above, this uncertainty depends on both the informational differences generated by the network and the precision of the regulator's public signal.

An increase in γ raises both D_i and D_j. Differentiating (117) with respect to γ yields

$$\frac{\partial V_{ij}}{\partial \gamma} = -\frac{2\beta^2}{D_j^3}\left(\frac{m_{ij}}{\beta} + \frac{m_{ij}^2}{D_i}\right) - \frac{\beta^2 m_{ij}^2}{D_j^2 D_i^2}. \tag{134}$$

Since all parameters are positive, it follows that

$$\frac{\partial V_{ij}}{\partial \gamma} < 0. \tag{135}$$

A more precise public signal therefore reduces the residual uncertainty that investor i faces about investor j's posterior belief. The regulator provides a common source of information about the institution's underlying condition, reducing the importance of differences in the private information available to investors through the network.

The two results above show that regulatory communication attenuates the role of network structure at both the first-order and higher-order levels. At the first-order level, a more precise public signal reduces the marginal weight placed on individual network

signals in investors' posterior beliefs. At the higher-order level, it reduces the uncertainty that investors face about one another's beliefs. Thus, as the regulator's public signal becomes more precise, decentralized differences in information play a progressively smaller role in both belief formation and strategic uncertainty.

The limiting case makes this mechanism particularly transparent. As the precision of the regulator's signal becomes arbitrarily large,

$$\gamma \to \infty, \tag{136}$$

we have

$$\frac{\partial \hat{\theta}_i}{\partial x_\ell} = \frac{\beta}{D_i} \longrightarrow 0, \tag{137}$$

and

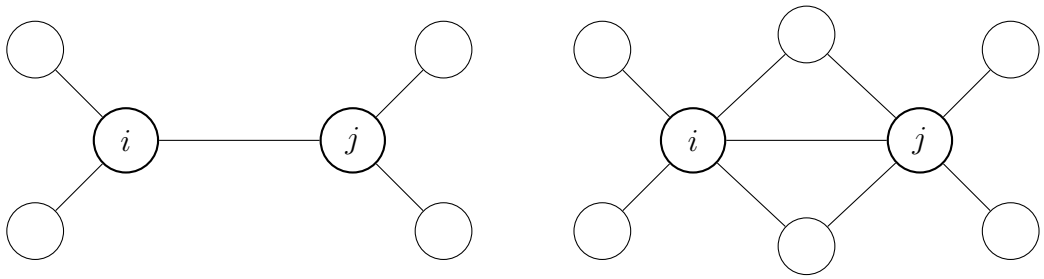
$$V_{ij} \longrightarrow 0. \tag{138}$$

In this limit, the common public signal dominates the decentralized information transmitted through the network. Individual private signals have a vanishing marginal influence on investors' posterior beliefs, while investors face vanishing uncertainty about how others assess the institution's underlying condition. As a result, the network ceases to be an important source of belief heterogeneity or higher-order uncertainty. Investors increasingly condition their funding decisions on the same common assessment of the institution, rather than on differences in information generated by their network positions. Regulatory communication therefore neutralizes the distinctive informational role of the network in the coordination problem.

4.4.3 Network Expansion and Informational Sparsity

We now examine how changes in the network affect the residual uncertainty that investor i faces about investor j's posterior belief. The effect of an additional network connection depends not simply on whether investor j becomes better informed, but also on whether the information acquired through that connection is observed by investor i.

The key distinction is captured by m_{ij}, which measures the number of private signals entering investor j's information set that are not observed by investor i. A new connection may therefore either provide investor j with information that is already available to i, or give j access to genuinely new information that remains unobserved by i. The two cases are illustrated in Figure 2.



(a) Locally sparse information

New neighbour not observed by i:
$\Delta k_j = 1, \quad \Delta m_{ij} = 1$

(b) Locally shared information

New neighbour already observed by i:
$\Delta k_j = 1, \quad \Delta m_{ij} = 0$

Figure 2: Network expansion and local informational overlap. When a new connection gives investor j access to a signal not observed by investor i, both k_j and m_{ij} increase. When the new signal is already observed by i, only k_j increases.

Expansion through shared information. First, suppose that a new connection gives investor j access to a private signal that is already observed by investor i. The additional connection therefore increases k_j while leaving m_{ij} unchanged. Holding the informational asymmetry between the two investors fixed, we have

$$\frac{\partial V_{ij}}{\partial k_j} = -\frac{2\beta^3}{D_j^3}\left(\frac{m_{ij}}{\beta} + \frac{m_{ij}^2}{D_i}\right) < 0. \tag{139}$$

Additional connectivity therefore unambiguously reduces higher-order uncertainty in this case. The new connection makes investor j better informed, but does not introduce any additional information whose realization is hidden from investor i. Consequently, investor i can predict investor j's posterior belief more accurately.

In the context of a potential bank run, this makes investors' informational environments more aligned. Investor i has a better understanding of the information on which investor j bases her withdrawal decision and can therefore form more precise expectations about whether j is likely to withdraw. Connectivity through shared information thus reduces uncertainty about others' beliefs and actions.

Expansion through unshared information. The situation differs when a new connection gives investor j access to a private signal that investor i does not observe. In this case, the additional connection simultaneously increases the amount of information available to j and the informational asymmetry between the two investors:

$$\Delta k_j = 1, \qquad \Delta m_{ij} = 1.$$

These two changes have opposing effects on higher-order uncertainty. The increase in k_j makes investor j's posterior more precise and therefore tends to reduce V_{ij}. By

contrast, the increase in m_{ij} means that a larger part of the information underlying j's posterior is not observed by i. Holding k_j fixed, this effect is

$$\frac{\partial V_{ij}}{\partial m_{ij}} = \frac{\beta^2}{D_j^2}\left(\frac{1}{\beta} + \frac{2m_{ij}}{D_i}\right) > 0. \tag{140}$$

The total effect of a new connection that introduces an unshared signal is therefore ambiguous. Using a local comparative-static decomposition,

$$dV_{ij} = \frac{\partial V_{ij}}{\partial k_j} dk_j + \frac{\partial V_{ij}}{\partial m_{ij}} dm_{ij}. \tag{141}$$

For a network expansion with

$$\Delta k_j = 1, \qquad \Delta m_{ij} = 1,$$

the corresponding local change is

$$\Delta V_{ij} = -\frac{2\beta^3}{D_j^3}\left(\frac{m_{ij}}{\beta} + \frac{m_{ij}^2}{D_i}\right) + \frac{\beta^2}{D_j^2}\left(\frac{1}{\beta} + \frac{2m_{ij}}{D_i}\right), \tag{142}$$

whose sign is generally ambiguous.

The economic implication is that more connectivity does not necessarily make investors' beliefs more similar or more predictable. What matters is the informational overlap created by new connections. A link that provides investor j with information already available to investor i makes j's beliefs easier for i to predict. By contrast, a link that provides j with genuinely new information has two opposing effects: it improves j's information about the fundamental while making the informational basis of j's withdrawal decision less transparent to i.

This distinction is particularly important in a bank-run environment. Social and professional networks do not merely determine how much information investors possess. They also determine how much investors know about the information on which others base their decisions. Greater connectivity can therefore either make investors' beliefs and actions more predictable, strengthening the strategic link between their decisions, or preserve strategic uncertainty by generating locally unshared information.

The central implication is that the relevant feature of the network is not connectivity alone, but the degree of informational overlap across investors. Regulatory communication interacts directly with this mechanism. A more precise public signal provides investors with a common informational anchor, reducing the relative importance of locally unshared private information. Centralized regulatory communication can therefore partially substitute for informational overlap within the network.

5 Conclusion

This thesis studies how the transmission and distribution of information shape coordination under strategic uncertainty. Motivated by financial fragility and investor withdrawal decisions, the analysis emphasizes that information affects coordination through more than its direct effect on beliefs about an institution's underlying condition. It also shapes what investors know about the beliefs and information of other investors. These higher-order beliefs are central to coordination: investors form expectations not only about the underlying condition, but also about how other investors assess that condition and how they are likely to act.

The first part of the thesis studies coarse regulatory communication. Its central result is that the precision of the information available to the regulator and the precision of the information ultimately transmitted to investors are distinct. Even when the regulator's underlying information becomes arbitrarily precise, equilibrium uniqueness may survive when that information is communicated only coarsely. A binary message partitions the state space without fully revealing the underlying condition, leaving residual uncertainty for private information and strategic uncertainty to play a role. The implications of regulatory communication therefore cannot be understood solely by asking how accurately the regulator observes the underlying state. How that information is transmitted matters as well. In settings of financial stress, regulatory communication may thus influence coordination not only through the information it reveals, but also through the uncertainty that it leaves unresolved.

The second part of the thesis studies decentralized information transmitted through a network of investors alongside a common public signal from the regulator. In this environment, coordination depends not only on the amount of information available, but also on how that information is distributed across investors. Network connections affect both investors' assessments of the institution's underlying condition and their ability to infer what other investors know. In particular, the effect of additional connections depends on whether they provide information that is already shared locally or introduce information that remains unobserved by others. Greater connectivity therefore need not mechanically reduce higher-order uncertainty. At the same time, a more precise public signal reduces the marginal influence of decentralized network information and makes investors less uncertain about how others assess the institution. Regulatory communication can therefore reshape the coordination problem by attenuating the informational role of the network.

Taken together, the two models highlight a common theme: information precision and information structure are distinct objects. In the first model, the regulator may possess highly precise information while communicating only a coarse summary of it. In the second, investors may have access to substantial private information, but its strategic consequences depend on how that information is distributed across the network and on what investors can infer about one another's information. The relationship between infor-

mation and coordination therefore depends fundamentally on who observes information, how it is communicated, and what remains uncertain about the beliefs of others.

Several extensions follow naturally from the framework. One important direction is to endogenize regulatory communication, allowing the regulator to choose what to disclose, how precisely to communicate it, or whether to communicate at all. This would shift the analysis from the consequences of a given communication policy to the strategic design of communication itself. A second extension is to endogenize communication between investors. I explored such a setup in which investors strategically choose whether to transmit their private signals. While the resulting posterior mean can be characterized using the unified skew-normal distribution, obtaining tractable expressions for higher-order beliefs proved considerably more difficult. Developing methods to address this challenge would allow the network's informational structure itself to emerge as an equilibrium outcome.

Further extensions include allowing the regulator to use richer message spaces and studying larger, more heterogeneous networks through numerical methods and simulations. Such analyses could examine how sparsity, clustering, and centrality interact with higher-order uncertainty, investor coordination, and financial fragility.

Overall, the thesis shows that communication is not merely a device for reducing uncertainty. By determining which information becomes common, which information remains decentralized, and what investors can infer about one another's beliefs, communication can itself shape coordination and collective behavior. This perspective is particularly relevant for financial institutions facing the risk of coordinated withdrawals, where regulatory communication may influence not only how investors assess the institution's underlying condition, but also how they anticipate the actions of others. The framework developed here provides a starting point for studying richer forms of regulatory communication, endogenous information transmission, and networked financial fragility.

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A Proofs for the Benchmark Model

A.1 Existence and Uniqueness

Recall the threshold equation

$$t^*(y) = \frac{1}{n} + \frac{n-1}{n}\Phi\left[\frac{(\alpha+\gamma)t^*(y) - \alpha\bar{\theta} - \gamma y}{\beta\sqrt{\frac{1}{\beta} + \frac{1}{\alpha+\beta+\gamma}}}\right]. \tag{143}$$

For notational convenience, define

$$s \equiv \beta\sqrt{\frac{1}{\beta} + \frac{1}{\alpha+\beta+\gamma}} = \frac{\sqrt{\beta}\sqrt{\alpha+2\beta+\gamma}}{\sqrt{\alpha+\beta+\gamma}}, \tag{144}$$

and

$$q \equiv \frac{\alpha+\gamma}{s} = \frac{(\alpha+\gamma)\sqrt{\alpha+\beta+\gamma}}{\sqrt{\beta}\sqrt{\alpha+2\beta+\gamma}}. \tag{145}$$

Rearranging (143) gives

$$\Phi^{-1}\left(\frac{nt^*-1}{n-1}\right) - qt^* + C(y) = 0, \tag{146}$$

where

$$C(y) \equiv \frac{\alpha\bar{\theta} + \gamma y}{s}. \tag{147}$$

Define

$$H(t^*; y) \equiv \Phi^{-1}\left(\frac{nt^*-1}{n-1}\right) - qt^* + C(y). \tag{148}$$

A solution must satisfy

$$t^* \in \left(\frac{1}{n}, 1\right).$$

As $t^* \downarrow 1/n$,

$$\Phi^{-1}\left(\frac{nt^*-1}{n-1}\right) \longrightarrow -\infty,$$

and hence

$$\lim_{t^*\downarrow 1/n} H(t^*; y) = -\infty.$$

Similarly, as $t^* \uparrow 1$,

$$\Phi^{-1}\left(\frac{nt^*-1}{n-1}\right) \longrightarrow +\infty,$$

so that

$$\lim_{t^*\uparrow 1} H(t^*; y) = +\infty.$$

Since H is continuous, the intermediate value theorem implies that there exists at least

one $t^*(y) \in (1/n, 1)$ satisfying (146). Thus, a monotone threshold equilibrium exists for every realization of y.

We next establish uniqueness. Differentiating (148) yields

$$H_{t^*}(t^*; y) = \frac{n}{n-1} \frac{1}{\phi\left(\Phi^{-1}\left(\frac{nt^*-1}{n-1}\right)\right)} - q. \tag{149}$$

Since

$$\max_{z \in \mathbb{R}} \phi(z) = \frac{1}{\sqrt{2\pi}},$$

and since

$$\Phi^{-1}\left(\frac{nt^* - 1}{n-1}\right)$$

ranges over the entire real line as t^* ranges over $(1/n, 1)$, the minimum of H_{t^*} is

$$\min_{t^* \in (1/n,1)} H_{t^*}(t^*; y) = \frac{n}{n-1}\sqrt{2\pi} - q. \tag{150}$$

Suppose first that

$$q \leq \frac{n}{n-1}\sqrt{2\pi}. \tag{151}$$

Then

$$H_{t^*}(t^*; y) \geq 0 \qquad \text{for all } t^* \in (1/n, 1).$$

Moreover, the derivative can equal zero at at most one point. Hence H is strictly increasing on $(1/n, 1)$. Since H goes from $-\infty$ to $+\infty$, it has exactly one zero. Therefore, the monotone threshold equilibrium is unique for every y.

It remains to establish the converse. Suppose that

$$q > \frac{n}{n-1}\sqrt{2\pi}. \tag{152}$$

Then, by (150), there exists an interval around the point satisfying

$$\Phi^{-1}\left(\frac{nt^* - 1}{n-1}\right) = 0$$

on which

$$H_{t^*}(t^*; y) < 0.$$

On the other hand,

$$H_{t^*}(t^*; y) \longrightarrow +\infty$$

as $t^* \downarrow 1/n$ or $t^* \uparrow 1$. Thus, H is increasing near both boundaries and decreasing over an interior interval. Consequently, H has a local maximum and a local minimum.

Importantly, y enters H only through the additive term $C(y)$. Since

$$C(y) = \frac{\alpha\bar{\theta}}{s} + \frac{\gamma}{s}y$$

and $\gamma/s > 0$, $C(y)$ ranges over the entire real line as y varies over the real line. Varying y therefore shifts the graph of H vertically without changing its shape.

Hence, there exists a realization of y for which the local maximum of $H(t^*; y)$ is positive while its local minimum is negative. For such a realization, continuity and the boundary limits imply that

$$H(t^*; y) = 0$$

has three distinct solutions. Thus, uniqueness cannot hold for every realization of the public signal when (151) is violated.

Combining the two directions gives
The monotone threshold equilibrium is unique for every y

$$\iff \frac{(\alpha+\gamma)\sqrt{\alpha+\beta+\gamma}}{\sqrt{\beta}\sqrt{\alpha+2\beta+\gamma}} \leq \frac{n}{n-1}\sqrt{2\pi}.$$

□

A.2 Global-Game Limit

We next characterize the limit as private information becomes arbitrarily precise. Let $\beta \to \infty$, while α, γ, and y are held fixed.

From the threshold equation,

$$t^*(y) = \frac{1}{n} + \frac{n-1}{n}\Phi\left[\frac{(\alpha+\gamma)t^*(y) - \alpha\bar{\theta} - \gamma y}{\beta\sqrt{\frac{1}{\beta} + \frac{1}{\alpha+\beta+\gamma}}}\right].$$

The denominator of the argument of Φ satisfies

$$\beta\sqrt{\frac{1}{\beta} + \frac{1}{\alpha+\beta+\gamma}} = \sqrt{\beta + \frac{\beta^2}{\alpha+\beta+\gamma}} \longrightarrow \infty \tag{153}$$

as $\beta \to \infty$. Since the numerator remains finite for any finite $t^*(y)$ and y, it follows that

$$\frac{(\alpha+\gamma)t^*(y) - \alpha\bar{\theta} - \gamma y}{\beta\sqrt{\frac{1}{\beta} + \frac{1}{\alpha+\beta+\gamma}}} \longrightarrow 0. \tag{154}$$

Since $\Phi(\cdot) \longrightarrow \Phi(0) = \frac{1}{2}$, taking the limit of the threshold equation gives

$$\lim_{\beta \to \infty} t^*(y) = \frac{1}{n} + \frac{n-1}{2n} = \frac{n+1}{2n}. \tag{155}$$

□

B Proofs for Coarse Communication Model

B.1 Derivation of the Extended Skew-Normal Posterior

Conditional on the private signal x_i, the posterior distribution of the fundamental is

$$\theta \mid x_i \sim N(\mu_i, \sigma_i^2), \qquad \mu_i = \frac{\tau_0 \bar{\theta} + \tau_x x_i}{\tau_0 + \tau_x}, \qquad \sigma_i^2 = \frac{1}{\tau_0 + \tau_x}. \tag{156}$$

After observing the regulator's message, Bayes' rule implies

$$p(\theta \mid x_i, m) \propto \frac{1}{\sigma_i} \phi\left(\frac{\theta - \mu_i}{\sigma_i}\right) \Phi\left(z \frac{\theta - d}{\sigma_R}\right). \tag{157}$$

Let $u = (\theta - \mu_i)/\sigma_i$. Then

$$p(u \mid x_i, m) \propto \phi(u)\Phi(\alpha_0 + \alpha u), \qquad \alpha = \frac{z\sigma_i}{\sigma_R}, \qquad \alpha_0 = \frac{z(\mu_i - d)}{\sigma_R}. \tag{158}$$

Using

$$\int_{-\infty}^{\infty} \phi(u)\Phi(a + bu)\, du = \Phi\left(\frac{a}{\sqrt{1 + b^2}}\right), \tag{159}$$

the normalized posterior is

$$p(u \mid x_i, m) = \frac{\phi(u)\Phi(\alpha_0 + \alpha u)}{\Phi(\tau)}, \qquad \tau = \frac{\alpha_0}{\sqrt{1 + \alpha^2}} = \frac{z(\mu_i - d)}{\sqrt{\sigma_R^2 + \sigma_i^2}}. \tag{160}$$

Defining

$$\delta = \frac{\alpha}{\sqrt{1 + \alpha^2}} = \frac{z\sigma_i}{\sqrt{\sigma_R^2 + \sigma_i^2}}, \tag{161}$$

it follows that

$$\theta \mid x_i, m \sim ESN(\mu_i, \sigma_i, \alpha, \tau). \tag{162}$$

B.2 Posterior Mean

For the standardized posterior,

$$E[u \mid x_i, m] = \zeta_1(\tau, \delta) = \delta \frac{\phi(\tau)}{\Phi(\tau)} = \delta\lambda(\tau), \tag{163}$$

where

$$\lambda(\tau) = \frac{\phi(\tau)}{\Phi(\tau)}. \tag{164}$$

Since $\theta = \mu_i + \sigma_i u$, the posterior mean is therefore

$$E[\theta \mid x_i, m] = \mu_i + \sigma_i \delta \lambda(\tau) \equiv \mu_i^E. \tag{165}$$

B.3 Derivative of the Posterior Mean

Recall that

$$\mu_i^E = \mu_i + \sigma_i \delta \lambda(\tau), \qquad \mu_i = \frac{\tau_0 \bar{\theta} + \tau_x x_i}{\tau_0 + \tau_x}, \tag{166}$$

where

$$\lambda(\tau) = \frac{\phi(\tau)}{\Phi(\tau)}, \qquad \delta = \frac{z\sigma_i}{\sqrt{\sigma_R^2 + \sigma_i^2}}, \qquad \tau = \frac{z(\mu_i - d)}{\sqrt{\sigma_R^2 + \sigma_i^2}}. \tag{167}$$

Since σ_i and δ are independent of x_i,

$$\frac{\partial \mu_i}{\partial x_i} = \frac{\tau_x}{\tau_0 + \tau_x}, \qquad \frac{\partial \tau}{\partial x_i} = \frac{z}{\sqrt{\sigma_R^2 + \sigma_i^2}} \frac{\tau_x}{\tau_0 + \tau_x}. \tag{168}$$

Applying the chain rule and using $z^2 = 1$ therefore gives

$$\frac{\partial \mu_i^E}{\partial x_i} = \frac{\tau_x}{\tau_0 + \tau_x} \left[1 + \frac{\sigma_i^2}{\sigma_R^2 + \sigma_i^2} \lambda'(\tau) \right]. \tag{169}$$

By Sampford (1953),

$$-1 < \lambda'(\tau) < 0, \qquad \forall \tau \in \mathbb{R}. \tag{170}$$

Since

$$0 < \frac{\sigma_i^2}{\sigma_R^2 + \sigma_i^2} < 1,$$

the term in brackets is strictly positive. Hence,

$$\frac{\partial \mu_i^E}{\partial x_i} > 0. \tag{171}$$

Thus, a higher private signal always increases the investor's posterior mean, even after conditioning on the regulator's coarse message.

B.3.1 Derivative of the Posterior Density

We next derive the effect of the private signal on the posterior density. Recall that

$$p(\theta \mid x_i, m) = \frac{1}{\sigma_i} \frac{\phi(u)\Phi(w)}{\Phi(\tau)},$$

where

$$u = \frac{\theta - \mu_i}{\sigma_i}, \qquad w = \alpha_0 + \alpha u.$$

Using the definitions of α_0 and α,

$$w = \frac{z(\mu_i - d)}{\sigma_R} + \frac{z\sigma_i}{\sigma_R}\frac{\theta - \mu_i}{\sigma_i} = \frac{z(\theta - d)}{\sigma_R}.$$

Hence, w is independent of x_i.

Taking logarithms and differentiating therefore gives

$$\frac{\partial}{\partial x_i}\log p(\theta \mid x_i, m) = -u\frac{\partial u}{\partial x_i} - \frac{\phi(\tau)}{\Phi(\tau)}\frac{\partial \tau}{\partial x_i}.$$

Since

$$\frac{\partial u}{\partial x_i} = -\frac{1}{\sigma_i}\frac{\partial \mu_i}{\partial x_i},$$

and

$$\frac{\partial \tau}{\partial x_i} = \frac{z}{\sigma_R\sqrt{1+\alpha^2}}\frac{\partial \mu_i}{\partial x_i},$$

we obtain

$$\frac{\partial p(\theta \mid x_i, m)}{\partial x_i} = p(\theta \mid x_i, m)\frac{\partial \mu_i}{\partial x_i}\left[\frac{u}{\sigma_i} - \frac{z}{\sigma_R\sqrt{1+\alpha^2}}\frac{\phi(\tau)}{\Phi(\tau)}\right].$$

Thus, the derivative of the posterior density takes the form of the posterior density multiplied by a simple score term.

We can now substitute (B.3.1) into the derivative of the value function:

$$\frac{\partial V(x_i, t, m)}{\partial x_i} = \frac{n-1}{n}\int \Phi\left(\frac{t-\theta}{\sigma_x}\right)\frac{\partial p(\theta \mid x_i, m)}{\partial x_i}\,d\theta - \frac{\partial \mu_i^E}{\partial x_i}.$$

Define

$$I_1 = \int \Phi\left(\frac{t-\theta}{\sigma_x}\right)p(\theta \mid x_i, m)\,d\theta$$

and

$$I_2 = \int u\,\Phi\left(\frac{t-\theta}{\sigma_x}\right)p(\theta \mid x_i, m)\,d\theta.$$

Then

$$\frac{\partial V(x_i, t, m)}{\partial x_i} = \frac{n-1}{n}\frac{\partial \mu_i}{\partial x_i}\left[\frac{I_2}{\sigma_i} - cI_1\right] - \frac{\partial \mu_i^E}{\partial x_i},$$

where

$$c = \frac{z}{\sigma_R\sqrt{1+\alpha^2}}\frac{\phi(\tau)}{\Phi(\tau)}.$$

The first integral is simply

$$I_1 = \Pr(a_j = 1 \mid x_i, m).$$

For the second, using

$$u = \frac{\theta - \mu_i}{\sigma_i} = \frac{\theta - \mu_i^E}{\sigma_i} + \frac{\mu_i^E - \mu_i}{\sigma_i},$$

we obtain

$$I_2 = \frac{1}{\sigma_i} \operatorname{Cov}\left(\theta, \Pr(a_j = 1 \mid \theta) | x_i, m\right) + \zeta_1(\tau, \delta) I_1.$$

Hence,

$$\frac{I_2}{\sigma_i} = \frac{1}{\sigma_i^2} \operatorname{Cov}\left(\theta, \Pr(a_j = 1 \mid \theta) | x_i, m\right) + \frac{\zeta_1(\tau, \delta)}{\sigma_i} I_1.$$

The two terms involving I_1 cancel exactly. Indeed,

$$\frac{\zeta_1(\tau, \delta)}{\sigma_i} = \frac{\delta}{\sigma_i} \frac{\phi(\tau)}{\Phi(\tau)} = \frac{z}{\sigma_R \sqrt{1 + \alpha^2}} \frac{\phi(\tau)}{\Phi(\tau)} = c.$$

Therefore,

$$\frac{\partial V(x_i, t, m)}{\partial x_i} = \frac{n-1}{n} \frac{\partial \mu_i}{\partial x_i} \frac{1}{\sigma_i^2} \operatorname{Cov}\left(\theta, \Pr(a_j = 1 \mid \theta) | x_i, m\right) - \frac{\partial \mu_i^E}{\partial x_i}. \tag{172}$$

Finally,

$$\Pr(a_j = 1 \mid \theta) = \Phi\left(\frac{t - \theta}{\sigma_x}\right)$$

is strictly decreasing in θ, implying

$$\operatorname{Cov}\left(\theta, \Pr(a_j = 1 \mid \theta) | x_i, m\right) < 0.$$

Since

$$\frac{\partial \mu_i}{\partial x_i} > 0, \qquad \frac{\partial \mu_i^E}{\partial x_i} > 0,$$

both terms in (172) are negative. Hence,

$$\frac{\partial V(x_i, t, m)}{\partial x_i} < 0.$$

B.4 Tail Behavior of Posterior Mean

To establish the existence of a threshold equilibrium, recall that the private signal is evaluated at the conjectured threshold, $x_i = t$. Hence, the Gaussian posterior mean is

$$\mu_i(t) = \frac{\tau_0 \bar{\theta} + \tau_x t}{\tau_0 + \tau_x}, \tag{173}$$

so that

$$\lim_{t \to -\infty} \mu_i(t) = -\infty, \qquad \lim_{t \to +\infty} \mu_i(t) = +\infty. \tag{174}$$

The posterior mean after observing the regulator's message can be written as

$$\mu_i^E(t,m) = \mu_i(t) + \sigma_i \delta \lambda(\tau), \tag{175}$$

where

$$\lambda(\tau) = \frac{\phi(\tau)}{\Phi(\tau)}, \qquad \delta = \frac{z\sigma_i}{\sqrt{\sigma_R^2 + \sigma_i^2}}, \qquad \tau = \frac{z(\mu_i(t) - d)}{\sqrt{\sigma_R^2 + \sigma_i^2}}. \tag{176}$$

We now characterize the two tails.

The upper tail. Suppose $t \to +\infty$, so that $\mu_i(t) \to +\infty$. If $z = 1$, then $\tau \to +\infty$ and

$$\lambda(\tau) \longrightarrow 0.$$

The extended skew-normal correction therefore vanishes, implying

$$\mu_i^E(t,m) \longrightarrow +\infty.$$

If $z = -1$, then $\tau \to -\infty$. Using the standard Mills-ratio asymptotic

$$\lambda(\tau) \sim -\tau, \qquad \tau \to -\infty,$$

we obtain

$$\begin{aligned} \mu_i^E(t,m) &\sim \mu_i(t) - \frac{\sigma_i^2}{\sigma_R^2 + \sigma_i^2}\big(\mu_i(t) - d\big) \\ &= \frac{\sigma_R^2}{\sigma_R^2 + \sigma_i^2}\mu_i(t) + \frac{\sigma_i^2}{\sigma_R^2 + \sigma_i^2} d. \end{aligned} \tag{177}$$

Since $\sigma_R^2 > 0$ and $\mu_i(t) \to +\infty$, it follows that

$$\mu_i^E(t,m) \longrightarrow +\infty.$$

Hence, irrespective of the regulator's message,

$$\lim_{t \to +\infty} \mu_i^E(t,m) = +\infty. \tag{178}$$

The lower tail. Now suppose $t \to -\infty$, so that $\mu_i(t) \to -\infty$. If $z = -1$, then $\tau \to +\infty$, and the extended skew-normal correction vanishes. Thus,

$$\mu_i^E(t,m) \longrightarrow -\infty.$$

If $z = 1$, then $\tau \to -\infty$. Applying the same Mills-ratio asymptotic yields

$$\begin{aligned}\mu_i^E(t,m) &\sim \mu_i(t) - \frac{\sigma_i^2}{\sigma_R^2+\sigma_i^2}\big(\mu_i(t) - d\big) \\ &= \frac{\sigma_R^2}{\sigma_R^2+\sigma_i^2}\mu_i(t) + \frac{\sigma_i^2}{\sigma_R^2+\sigma_i^2}d. \end{aligned} \tag{179}$$

Since $\mu_i(t) \to -\infty$, this implies

$$\mu_i^E(t,m) \longrightarrow -\infty.$$

Therefore,

$$\lim_{t\to-\infty} \mu_i^E(t,m) = -\infty. \tag{180}$$

Combining these limits with the bounds on the coordination component,

$$\frac{1}{n} - \mu_i^E(t,m) \le G(t,m) \le 1 - \mu_i^E(t,m),$$

gives

$$\lim_{t\to-\infty} G(t,m) = +\infty, \qquad \lim_{t\to+\infty} G(t,m) = -\infty. \tag{181}$$

B.5 Derivative w.r.t the Threshold

We now derive the expression for the derivative of the value function with respect to the conjectured threshold. Define

$$J(t,x_i,m) = \int \phi\left(\frac{t-\theta}{\sigma_x}\right) p(\theta \mid x_i, m)\, d\theta.$$

Using the ESN representation of the posterior,

$$J(t,x_i,m) = \frac{1}{\Phi(\tau)\sigma_i}\int \phi\left(\frac{t-\theta}{\sigma_x}\right)\phi\left(\frac{\theta-\mu_i}{\sigma_i}\right)\Phi(w)\, d\theta,$$

where

$$w = \alpha_0 + \alpha\frac{\theta-\mu_i}{\sigma_i}.$$

Define

$$f_x(\theta) = \frac{1}{\sigma_x}\phi\left(\frac{t-\theta}{\sigma_x}\right), \qquad f_i(\theta) = \frac{1}{\sigma_i}\phi\left(\frac{\theta-\mu_i}{\sigma_i}\right).$$

Their product satisfies

$$f_x(\theta) f_i(\theta) = Z f_*(\theta),$$

where f_* is a Gaussian density with

$$\mu^* = \frac{\sigma_i^2 t + \sigma_x^2 \mu_i}{\sigma_x^2 + \sigma_i^2}, \qquad (\sigma^*)^2 = \frac{\sigma_x^2 \sigma_i^2}{\sigma_x^2 + \sigma_i^2},$$

and

$$Z = \frac{1}{\sqrt{2\pi(\sigma_x^2 + \sigma_i^2)}} \exp\left\{ -\frac{(t-\mu_i)^2}{2(\sigma_x^2 + \sigma_i^2)} \right\}.$$

Hence,

$$\frac{\partial V(x_i, t, m)}{\partial t} = \frac{n-1}{n} \frac{Z}{\Phi(\tau)} \int f_*(\theta) \Phi(w)\, d\theta.$$

Let

$$z^* = \frac{\theta - \mu^*}{\sigma^*}.$$

Then

$$w = A + B z^*,$$

where

$$A = \alpha_0 + \alpha \frac{\mu^* - \mu_i}{\sigma_i}, \qquad B = \alpha \frac{\sigma^*}{\sigma_i}.$$

Therefore,

$$\frac{\partial V(x_i, t, m)}{\partial t} = \frac{n-1}{n} \frac{Z}{\Phi(\tau)} \int \phi(z^*) \Phi(A + B z^*)\, dz^*.$$

Applying the identity

$$\int_{-\infty}^{\infty} \phi(z) \Phi(A + Bz)\, dz = \Phi\left(\frac{A}{\sqrt{1 + B^2}} \right),$$

gives

$$\frac{\partial V(x_i, t, m)}{\partial t} = \frac{n-1}{n} \frac{Z}{\Phi(\tau)} \Phi\left(\frac{A}{\sqrt{1 + B^2}} \right).$$

Substituting for A, B, μ^*, and σ^* yields

$$\frac{\partial V(x_i, t, m)}{\partial t} = \frac{n-1}{n} \frac{1}{\Phi(\tau)} \phi_{\sqrt{\sigma_x^2 + \sigma_i^2}}(t - \mu_i) \Phi\left(\frac{\alpha_0 + \alpha \frac{\sigma_i (t - \mu_i)}{\sigma_x^2 + \sigma_i^2}}{\sqrt{1 + \alpha^2 \frac{\sigma_x^2}{\sigma_x^2 + \sigma_i^2}}} \right).$$

Since $\Phi(\cdot) \leq 1$ and

$$\phi_\sigma(x) \leq \frac{1}{\sqrt{2\pi}\sigma},$$

it follows that

$$\frac{\partial V(x_i, t, m)}{\partial t} \leq \frac{n-1}{n} \frac{1}{\sqrt{2\pi(\sigma_x^2 + \sigma_i^2)} \Phi(\tau)}.$$

B.6 Limiting Cases of the Sufficiency Condition

We examine the sufficient condition in the limits $\sigma_R \to \infty$ and $\sigma_R \to 0$.

B.6.1 Uninformative Regulator: $\sigma_R \to \infty$

Recall that

$$\frac{\partial \mu_i^E}{\partial x_i} = \frac{\tau_x}{\tau_0 + \tau_x} + \sigma_i \zeta_1'(\tau, \delta) \frac{z\tau_x}{\sigma_R \sqrt{1 + \alpha^2(\tau_0 + \tau_x)}}. \tag{182}$$

As $\sigma_R \to \infty$,

$$\alpha = \frac{z\sigma_i}{\sigma_R} \longrightarrow 0, \qquad \tau \longrightarrow 0, \tag{183}$$

so the second term converges to zero. Hence,

$$\frac{\partial \mu_i^E}{\partial x_i} \longrightarrow \frac{\tau_x}{\tau_0 + \tau_x} = \frac{\sigma_i^2}{\sigma_x^2}. \tag{184}$$

Moreover, since $\Phi(\tau) \to \Phi(0) = 1/2$,

$$\frac{n-1}{n} \frac{1}{\sqrt{2\pi(\sigma_x^2 + \sigma_i^2)}\Phi(\tau)} \longrightarrow \frac{n-1}{n} \sqrt{\frac{2}{\pi(\sigma_x^2 + \sigma_i^2)}}. \tag{185}$$

Therefore, the sufficient condition becomes

$$\frac{\sigma_i^2}{\sigma_x^2} \geq \frac{n-1}{n} \sqrt{\frac{2}{\pi(\sigma_x^2 + \sigma_i^2)}}. \tag{186}$$

B.6.2 Perfectly Informed but Coarse Regulator: $\sigma_R \to 0$

Now consider $\sigma_R \to 0$. The regulator then observes the fundamental arbitrarily precisely while communicating only the binary message

$$m = \mathbb{1}\{\theta \geq d\}.$$

The limiting posterior is therefore truncated normal, with

$$m = 1 \Longleftrightarrow \theta \geq d, \qquad m = 0 \Longleftrightarrow \theta < d.$$

Since

$$\alpha = \frac{z\sigma_i}{\sigma_R}, \qquad \delta = \frac{\alpha}{\sqrt{1 + \alpha^2}}, \tag{187}$$

we have

$$\alpha \longrightarrow \pm\infty, \qquad \delta \longrightarrow z.$$

Defining

$$\tau = \frac{z(\mu_i - d)}{\sigma_i}, \qquad \lambda(\tau) = \frac{\phi(\tau)}{\Phi(\tau)}, \tag{188}$$

the limiting posterior mean is

$$\mu_i^E \longrightarrow \mu_i + z\sigma_i\lambda(\tau). \tag{189}$$

Consequently,

$$\frac{\partial \mu_i^E}{\partial x_i} \longrightarrow \frac{\sigma_i^2}{\sigma_x^2}\left[1 - \lambda(\tau)\left(\tau + \lambda(\tau)\right)\right]. \tag{190}$$

Since

$$\sqrt{\sigma_R^2 + \sigma_i^2} \longrightarrow \sigma_i, \qquad \tau \longrightarrow \frac{z(\mu_i - d)}{\sigma_i},$$

the sufficient condition becomes

$$\frac{\sigma_i^2}{\sigma_x^2}\left[1 - \lambda(\tau)\left(\tau + \lambda(\tau)\right)\right] \geq \frac{n-1}{n}\frac{1}{\sqrt{2\pi(\sigma_x^2 + \sigma_i^2)}\Phi(\tau)}, \tag{191}$$

where

$$\tau = \frac{z(\mu_i - d)}{\sigma_i}. \tag{192}$$

Equivalently,

$$\sqrt{2\pi(\sigma_x^2 + \sigma_i^2)}\frac{\sigma_i^2}{\sigma_x^2}\Phi(\tau)\left[1 - \lambda(\tau)\left(\tau + \lambda(\tau)\right)\right] \geq \frac{n-1}{n}. \tag{193}$$

B.7 Derivation of the Coordination Probability

Recall that

$$V(x_i, t, m) = \frac{1}{n} + \frac{n-1}{n}\Pr(x_j < t \mid x_i, m) - E[\theta \mid x_i, m]. \tag{194}$$

Conditional on x_i, the posterior before observing the regulator's message is

$$\theta \mid x_i \sim N(\mu_i, \sigma_i^2), \tag{195}$$

where

$$\sigma_i^2 = \left(\frac{1}{\sigma_\theta^2} + \frac{1}{\sigma_x^2}\right)^{-1}, \qquad \mu_i = \sigma_i^2\left(\frac{\bar{\theta}}{\sigma_\theta^2} + \frac{x_i}{\sigma_x^2}\right). \tag{196}$$

Define

$$S_x = \sqrt{\sigma_x^2 + \sigma_i^2}, \qquad S_R = \sqrt{\sigma_R^2 + \sigma_i^2}, \tag{197}$$

and

$$z = \begin{cases} 1, & m = 1, \\ -1, & m = 0. \end{cases} \tag{198}$$

The posterior mean after observing the regulator's message is

$$\mu_i^E = \mu_i + \frac{z\sigma_i^2}{S_R}\frac{\phi(\tau)}{\Phi(\tau)}, \qquad \tau = \frac{z(\mu_i - d)}{S_R}. \tag{199}$$

Conditional on x_i, the pair (x_j, y) is jointly Gaussian. Setting

$$a = \frac{t - \mu_i}{S_x}, \qquad \rho = \frac{z\sigma_i^2}{S_x S_R}, \tag{200}$$

the coordination probability is

$$\Pr(x_j < t \mid x_i, m) = \frac{\Phi_2(a, \tau; -\rho)}{\Phi(\tau)}. \tag{201}$$

Substituting these expressions into the payoff difference yields

$$V(x_i, t, m) = \frac{1}{n} + \frac{n-1}{n}\frac{\Phi_2\left(\frac{t-\mu_i}{S_x}, \frac{z(\mu_i - d)}{S_R}; -\frac{z\sigma_i^2}{S_x S_R}\right)}{\Phi\left(\frac{z(\mu_i - d)}{S_R}\right)} - \mu_i - \frac{z\sigma_i^2}{S_R}\frac{\phi\left(\frac{z(\mu_i - d)}{S_R}\right)}{\Phi\left(\frac{z(\mu_i - d)}{S_R}\right)}. \tag{202}$$

B.8 Limiting Case: Uninformative Regulator

As $\sigma_R \to \infty$,

$$S_R = \sqrt{\sigma_R^2 + \sigma_i^2} \longrightarrow \infty, \qquad \tau(t) = \frac{z(\mu_i(t) - d)}{S_R} \longrightarrow 0, \qquad \rho = \frac{z\sigma_i^2}{S_x S_R} \longrightarrow 0. \tag{203}$$

Moreover,

$$\frac{z\sigma_i^2}{S_R}\frac{\phi(\tau(t))}{\Phi(\tau(t))} \longrightarrow 0, \tag{204}$$

and hence

$$\mu_i^E(t, m) \longrightarrow \mu_i(t). \tag{205}$$

For the coordination term,

$$\begin{aligned} \frac{\Phi_2(a(t), \tau(t); -\rho)}{\Phi(\tau(t))} &\longrightarrow \frac{\Phi_2(a(t), 0; 0)}{\Phi(0)} \\ &= \frac{\Phi(a(t))\Phi(0)}{\Phi(0)} = \Phi(a(t)). \end{aligned} \tag{206}$$

Therefore, the threshold equation converges to

$$\frac{1}{n} + \frac{n-1}{n}\Phi\left(\frac{t - \mu_i(t)}{\sqrt{\sigma_x^2 + \sigma_i^2}}\right) - \mu_i(t) = 0. \tag{207}$$

The regulator's message is therefore uninformative in the limit, so the threshold is independent of both m and d. Denoting the limiting threshold by t^{NR},

$$t_1^* = t_0^* = t^{NR}. \tag{208}$$

Thus, as $\sigma_R \to \infty$, the model converges to the benchmark in which agents rely exclusively on their private signals.

B.9 Limiting Case: $\sigma_R \to 0$

As $\sigma_R \to 0$, the regulator's signal

$$y = \theta + \varepsilon_R, \qquad \varepsilon_R \sim N(0, \sigma_R^2), \tag{209}$$

converges to θ. Since

$$m = \mathbf{1}\{y \geq d\}, \tag{210}$$

the limiting message is

$$m = \begin{cases} 1, & \theta \geq d, \\ 0, & \theta < d. \end{cases} \tag{211}$$

Thus, the regulator perfectly reveals which side of the cutoff d the fundamental lies on.

Moreover,

$$S_R \longrightarrow \sigma_i, \qquad \tau(t) \longrightarrow \frac{z(\mu_i(t) - d)}{\sigma_i}, \qquad \rho \longrightarrow \frac{z\sigma_i}{\sqrt{\sigma_x^2 + \sigma_i^2}}. \tag{212}$$

Hence, the posterior mean converges to

$$\mu_i^E(t, m) \longrightarrow \mu_i(t) + z\sigma_i \frac{\phi\left(\dfrac{z(\mu_i(t) - d)}{\sigma_i}\right)}{\Phi\left(\dfrac{z(\mu_i(t) - d)}{\sigma_i}\right)}. \tag{213}$$

The limiting posterior is therefore the Gaussian posterior $N(\mu_i(t), \sigma_i^2)$ truncated below d when $m = 1$ and above d when $m = 0$.

Similarly, the coordination term converges to

$$\frac{\Phi_2\left(a(t), \dfrac{z(\mu_i(t) - d)}{\sigma_i}; -\dfrac{z\sigma_i}{\sqrt{\sigma_x^2 + \sigma_i^2}}\right)}{\Phi\left(\dfrac{z(\mu_i(t) - d)}{\sigma_i}\right)}. \tag{214}$$

Hence, the limiting threshold is characterized by

$$\begin{aligned} 0 = \frac{1}{n} + \frac{n-1}{n} \frac{\Phi_2\left(a(t), \dfrac{z(\mu_i(t)-d)}{\sigma_i}; -\dfrac{z\sigma_i}{\sqrt{\sigma_x^2+\sigma_i^2}}\right)}{\Phi\left(\dfrac{z(\mu_i(t)-d)}{\sigma_i}\right)} \\ - \mu_i(t) - z\sigma_i \frac{\phi\left(\dfrac{z(\mu_i(t)-d)}{\sigma_i}\right)}{\Phi\left(\dfrac{z(\mu_i(t)-d)}{\sigma_i}\right)}. \end{aligned} \tag{215}$$

B.10 Comparative Statics w.r.t the Regulatory Cutoff

For completeness, we provide the derivation of the comparative static result in Section 3.8. Holding t fixed, the cutoff d enters the coordination probability only through

$$\tau(t,d) = \frac{z\left(\mu_i(t)-d\right)}{\sqrt{\sigma_R^2+\sigma_i^2}}, \tag{216}$$

while

$$a(t) = \frac{t-\mu_i(t)}{\sqrt{\sigma_x^2+\sigma_i^2}}, \qquad \rho = \frac{z\sigma_i^2}{\sqrt{\sigma_x^2+\sigma_i^2}\sqrt{\sigma_R^2+\sigma_i^2}} \tag{217}$$

are independent of d. Hence,

$$\frac{\partial \tau}{\partial d} = -\frac{z}{\sqrt{\sigma_R^2+\sigma_i^2}}. \tag{218}$$

The coordination probability is

$$F(t,d;m) = \frac{\Phi_2\left(a,\tau;-\rho\right)}{\Phi(\tau)}. \tag{219}$$

Using

$$\frac{\partial}{\partial \tau}\Phi_2(a,\tau;-\rho) = \phi(\tau)\Phi\left(\frac{a+\rho\tau}{\sqrt{1-\rho^2}}\right), \tag{220}$$

we obtain

$$\frac{\partial F}{\partial \tau} = \frac{\phi(\tau)}{\Phi(\tau)}\left[\Phi\left(\frac{a+\rho\tau}{\sqrt{1-\rho^2}}\right) - F(t,d;m)\right]. \tag{221}$$

To determine the sign with respect to d, note that conditional on $x_i = t$,

$$\mathrm{Cov}(x_j, y \mid x_i = t) = \sigma_i^2 > 0. \tag{222}$$

For $m = 1$, increasing d shifts the conditioning event $y \geq d$ towards higher realizations of y, which are positively associated with higher realizations of x_j. For $m = 0$, increasing d relaxes the event $y < d$ and admits higher realizations of y, again shifting the conditional

distribution of x_j upward. In both cases, the probability that $x_j < t$ falls:

$$\frac{\partial F(t,d;m)}{\partial d} < 0, \qquad m \in \{0,1\}. \tag{223}$$

Next, write the posterior mean as

$$\mu_i^E(t,d;m) = \mu_i(t) + \frac{z\sigma_i^2}{\sqrt{\sigma_R^2+\sigma_i^2}}\lambda(\tau), \tag{224}$$

where

$$\lambda(\tau) = \frac{\phi(\tau)}{\Phi(\tau)}, \qquad \lambda'(\tau) = -\lambda(\tau)\left[\tau + \lambda(\tau)\right]. \tag{225}$$

Using (218),

$$\begin{aligned} \frac{\partial \mu_i^E}{\partial d} &= \frac{z\sigma_i^2}{\sqrt{\sigma_R^2+\sigma_i^2}}\lambda'(\tau)\frac{\partial \tau}{\partial d} \\ &= \frac{\sigma_i^2}{\sigma_R^2+\sigma_i^2}\lambda(\tau)\left[\tau+\lambda(\tau)\right] > 0. \end{aligned} \tag{226}$$

Finally, the equilibrium threshold satisfies

$$G(t_m^*, d; m) = 0. \tag{227}$$

By the implicit function theorem,

$$\frac{dt_m^*}{dd} = -\frac{G_d(t_m^*, d; m)}{G_t(t_m^*, d; m)}. \tag{228}$$

Since

$$G_d = \frac{n-1}{n}\frac{\partial F}{\partial d} - \frac{\partial \mu_i^E}{\partial d} < 0, \tag{229}$$

and, under the uniqueness condition,

$$G_t < 0, \tag{230}$$

it follows that

$$\frac{dt_m^*}{dd} < 0. \tag{231}$$